Online Appendix for:
From Dual to Unified Employment Protection: Transition and Steady State

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Contents

C Computational Details

C.1 Computing Steady States

We omit the time subscript in this subsection to indicate that the economy is in steady state. Computing a steady state is not a trivial task because $U^y(\Delta,\tau)$, $W^y(z,T)$, $W^o(z,T)$, $J^y(z,T)$, $J^o(z,T)$, as well as $w^y(z,T)$, $w^o(z,T)$, can only be recovered by solving fixed-point problems. Our algorithm is as follows:

1. Solve for $W^o(z,T)$, $J^o(z,T)$, $w^o(z,T)$ using the following steps:

   (a) Set initial guesses $\widehat{W}^o(z,T)$, $\widehat{J}^o(z,T)$, $\widehat{w}^o(z,T)$, where we use $\widehat{\cdot}$ to indicate a guess.

   (b) Compute the reservation wage of the worker $\underline{w}^o(z,T)$ and that of the firm $\overline{w}^o(z,T)$ associated with $\widehat{W}^o(z,T)$ and $\widehat{J}^o(z,T)$ using equations (17) and (18).

   (c) If $\underline{w}^o(z,T) \le \overline{w}^o(z,T)$, then solve for the wage w using the first-order condition of the generalised Nash product:

$$\frac{\beta}{1+\kappa_t}\left(z-(1+\kappa)w+\frac{1-\chi}{1+r}\sum_{z'}\pi_{z,z'}\max\left\{\widehat{J}^o(z',T),-\Phi(T)\right\}+\Phi(T)\right)$$
$$=\frac{1-\beta}{u'(w)}\left(u(w)+\frac{1-\chi}{1+r}\sum_{z'}\pi_{z,z'}\max\left\{\widehat{W}^o(z,T),U^o(T)\right\}-U^o(T)\right)$$

   and update $\widehat{w}^o(z,T)$ using this value (observe that $U^o(T)$ is pinned down by equation (6)). This first-order condition is a non-linear equation that can be solved using, e.g., the bisection method. If $\overline{w}^o(z,T) < \underline{w}^o(z,T)$, set $\widehat{w}^o(z,T) = \frac{1}{2}\left(\overline{w}^o(z,T)+\underline{w}^o(z,T)\right)$.

   (d) Update $\widehat{W}^o(z,T)$, $\widehat{J}^o(z,T)$ using equations (9) and (11).

   (e) If initial and updated guesses for value functions and wages are close enough, then we are done. Otherwise, go back to step (1a).

2. Compute $W^o(z,\tau)$, $J^o(z,\tau)$, $w^o(z,\tau)$ recursively from $\tau = T$. That is:

   (a) Compute the reservation wage of the worker $\underline{w}^o(z,\tau)$ and that of the firm $\overline{w}^o(z,\tau)$ using equations (17) and (18). Notice that the continuation values only involve $\tau+1$, which allows to compute $\underline{w}^o(z,\tau)$ and $\overline{w}^o(z,\tau)$.

   (b) If $\underline{w}^o(z,\tau) \le \overline{w}^o(z,\tau)$, then solve for the Nash-bargained wage using the first-order condition (14). The continuation values in this equation depend on $\tau+1$ only, and the outside option of the worker $U^o(\tau)$ is pre-determined.

   (c) Compute the value functions $W^o(z,\tau)$ and $J^o(z,\tau)$ from equations (9) and (11).

3. Solve for $U^y(\Delta,\tau)$, $W^y(z,\tau)$, $J^y(z,\tau)$, $w^y(z,\tau)$ using the following steps:

   (a) Set an initial guess for $\widehat{U}^y(\Delta,\tau)$.

(b) Solve for $W^y(z,T)$, $J^y(z,T)$, $w^y(z,T)$ using a methodology similar to step (1), i.e.: (i) Set initial guesses $\widehat{W}^y(z,T)$, $\widehat{J}^y(z,T)$, $\widehat{w}^y(z,T)$; (ii) Use equations (16) and (18) to obtain the reservation wages $\underline{w}^y(z,T)$ and $\overline{w}^y(z,T)$ implied by $\widehat{W}^y(z,T)$ and $\widehat{J}^y(z,T)$; (iii) Use the analogue of step (1c) to update the wage. Observe that $\widehat{U}^y(\Delta,T)$ is used as the outside option of the worker in the Nash bargain; (iv) Update $\widehat{W}^y(z,T)$ and $\widehat{J}^y(z,T)$ using equations (8) and (10); (v) Iterate until convergence.

(c) Compute $W^y(z,\tau)$, $J^y(z,\tau)$, $w^y(z,\tau)$ recursively from $\tau=T$ using a methodology similar to step (2). Again, observe that knowledge of $\widehat{U}^y(\Delta,\tau)$ is required to compute the Nash-bargained wage.

(d) Use the Bellman equation of a young unemployed worker to update $\widehat{U}^y(\Delta,\tau)$. If initial and updated guesses are close enough, then we are done. Otherwise, go back to step (3a) using the updated $\widehat{U}^y(\Delta,\tau)$.

The algorithm builds on the observation that, in a steady state, the asset values $U^y(\Delta,\tau)$, $W^y(z,T)$, $W^o(z,T)$, $J^y(z,T)$ and $J^o(z,T)$ are solutions to an infinite-horizon problem, whereas $W^y(z,\tau)$, $W^o(z,\tau)$, $J^y(z,\tau)$, $J^o(z,\tau)$ for all $\tau<T$ solve a standard finite-period (T) problem, and $U^o(\tau)$ is completely determined.

A steady state also involves finding the equilibrium tuple (θ,κ) and the expected duration of a jobless spell Δ. Therefore, the algorithm above is nested into two outer loops to iterate on the tuple (θ,κ). First, we fix the payroll tax κ and iterate to solve for labour market tightness θ. At a given θ, the expected duration Δ is fixed and known since the economy is at a steady state (see equation (B4) in Appendix B). Second, we solve for the time-invariant distribution, calculate the budget-clearing payroll tax and update κ accordingly. Finally, notice that the severance pay function $\phi(\tau)$ is specified as a function of the average wage $\widetilde{w}$. Since this is an equilibrium object, we must add an outer loop to iterate on $\widetilde{w}$.

C.2 Computing Transition Paths

The transition path eliminates the infinite horizon problem analysed in Appendix C.1 because all continuation values depend on $t+1$. The other key observation is that the computation needs not keep track of all the sequences used to define the transition path (cf. Definition 2). The 'only' required objects are: the cross-sectional distribution of agents at t_0, the sequences $(W_t^y(z_0,0,1))_{t=t_0,\dots,t_1}$, $(w_t^y(z,\tau,\epsilon),\ w_t^o(z,\tau,\epsilon))_{t=t_0,\dots,t_1}$, $(\overline{z}_t^y(\tau,\epsilon),\ \overline{z}_t^o(\tau,\epsilon))_{t=t_0,\dots,t_1}$ and $(\theta_t)_{t=t_0,\dots,t_1}$, as well as the time path $(\kappa_t)_{t=t_0,\dots,t_1}$. In these notations, in line with Proposition 2, we introduce an additional state variable $\epsilon\in\{0,1\}$ indicating whether the worker-firm pair already exists when the reform is introduced ($\epsilon=0$) or not ($\epsilon=1$, which results in the ϕ_1 function in equation (22)). Then, our algorithm works as follows:

1. Compute the equilibrium allocation of the economy in period t_1.

2. Guess a time path for the payroll tax $(\widehat{\kappa}_t)_{t=t_0,\dots,t_1}$.

3. Solve for value functions, wages, separation decisions and labour market tightness backwards from t_1 until t_0 as follows:

(a) Compute the severance pay function $\phi_t(\tau)$ for workers in $\epsilon = 0$ using Proposition 2.

(b) Compute market tightness θ_t consistent with free entry at time t, and store it.

(c) Use Proposition 1 to compute $U_t^y(\Delta_t, \tau)$ and $U_{t+1}^y(\Delta_{t+1}, \tau)$. Notice that these require the sequences of Δ_t and $W_{t+1}^y(z_0, 0, 1)$ from t onwards, which we have at hand.

(d) Solve for the wage functions $w_t^y(z, \tau, \epsilon)$ and $w_t^o(z, \tau, \epsilon)$ at time t, store them, and compute the asset values of employment. Finally, compute the job separation decisions $\overline{z}_t^y(\tau, \epsilon)$ and $\overline{z}_t^o(\tau, \epsilon)$ at time t and store them.

4. Initialize the distribution using the cross-sectional distribution of agents at t_0.

5. Using $(\theta_t)_{t=t_0,\ldots,t_1}$, $(w_t^y(z, \tau, \epsilon),\ w_t^o(z, \tau, \epsilon))_{t=t_0,\ldots,t_1}$ and $(\overline{z}_t^y(\tau, \epsilon),\ \overline{z}_t^o(\tau, \epsilon))_{t=t_0,\ldots,t_1}$ and the stock-flow equations (A1)–(A7) (augmented to include the state variable ϵ), compute the evolution of the cross-sectional distribution from t_0 until t_1 . Each period, compute the budget-clearing value of the payroll tax κ_t to obtain $(\kappa_t)_{t=t_0,\ldots,t_1}$.

6. If $(\widehat{\kappa}_t)_{t=t_0,\ldots,t_1}$ and $(\kappa_t)_{t=t_0,\ldots,t_1}$ are close enough, then we are done. Otherwise, go back to step (2) with a new guess.

To ensure that the payroll tax obtained at the end of the transition path coincides with the t_1 payroll tax, we allow for a very large number of periods between t_0 and t_1. In our applications, we set the number of period to 1,000 (250 years). After 500 periods, the measure of workers who remain in state $\epsilon = 0$ is 0.0001.

D Additional Tables and Figures

D.1 Welfare Effects with Savings

Table D1 presents the welfare effects of the unified EPL scheme in the approximate model with savings. Since the transition path of this model is too costly to compute, welfare changes for those who are employed at the time of the reform are based on steady-state approximations. These calculations are nevertheless informative because the EPL transition in our model is quickly completed.

Table D1. Welfare effects in the approximate model with savings

	Welfare change	**Asset change**
new entrants	1.26	–
average, all workers	0.31	14.6
average, young workers	0.43	18.2
average, older workers	-0.73	-0.40

Notes: The table reports steady-state welfare changes (measured in consumption-equivalent units) and steady-state changes in asset levels from introducing a unified EPL in the approximate model with savings. All entries are expressed in percent.

The first remark concerns changes in the steady-state welfare of newborn agents – the only 'legitimate' criterion for steady-state comparisons. We find that the benchmark model overestimates the welfare gain of reforming EPL only slightly (1.52 percent in Table 4 vs. in 1.26 percent Table D1). Second, average welfare losses among older workers are in the same ballpark (-0.73 vs. -0.79 in Table 5 describing the benchmark model). Third, by contrast, the welfare gain among young workers is smaller in the approximate model (0.43 vs. 1.19 in the benchmark model). As the rightmost column (displaying changes in asset levels) shows, the EPL reform induces young workers to build up additional wealth by saving a larger share of their income, at the expense of lower consumption. The benchmark model ignores this effect.

D.2 Welfare Effects under a *Statu-quo* Reform

Table D2 presents the welfare effects of the transition towards unified EPL under a statu-quo reform. Figure D1 shows the time path of several variables during the transition. See the main text for a discussion.

Table D2. Welfare effects under a *statu-quo* reform

		Average in each quintile				
	Average	**1st**	**2nd**	**3rd**	**4th**	**5th**
young workers	1.29	1.16	1.21	1.27	1.35	1.48
older workers	0.21	0.15	0.19	0.22	0.23	0.24

Notes: The table reports the welfare changes (measured in consumption-equivalent units) arising from the transition towards unified EPL. All entries are expressed in percent.

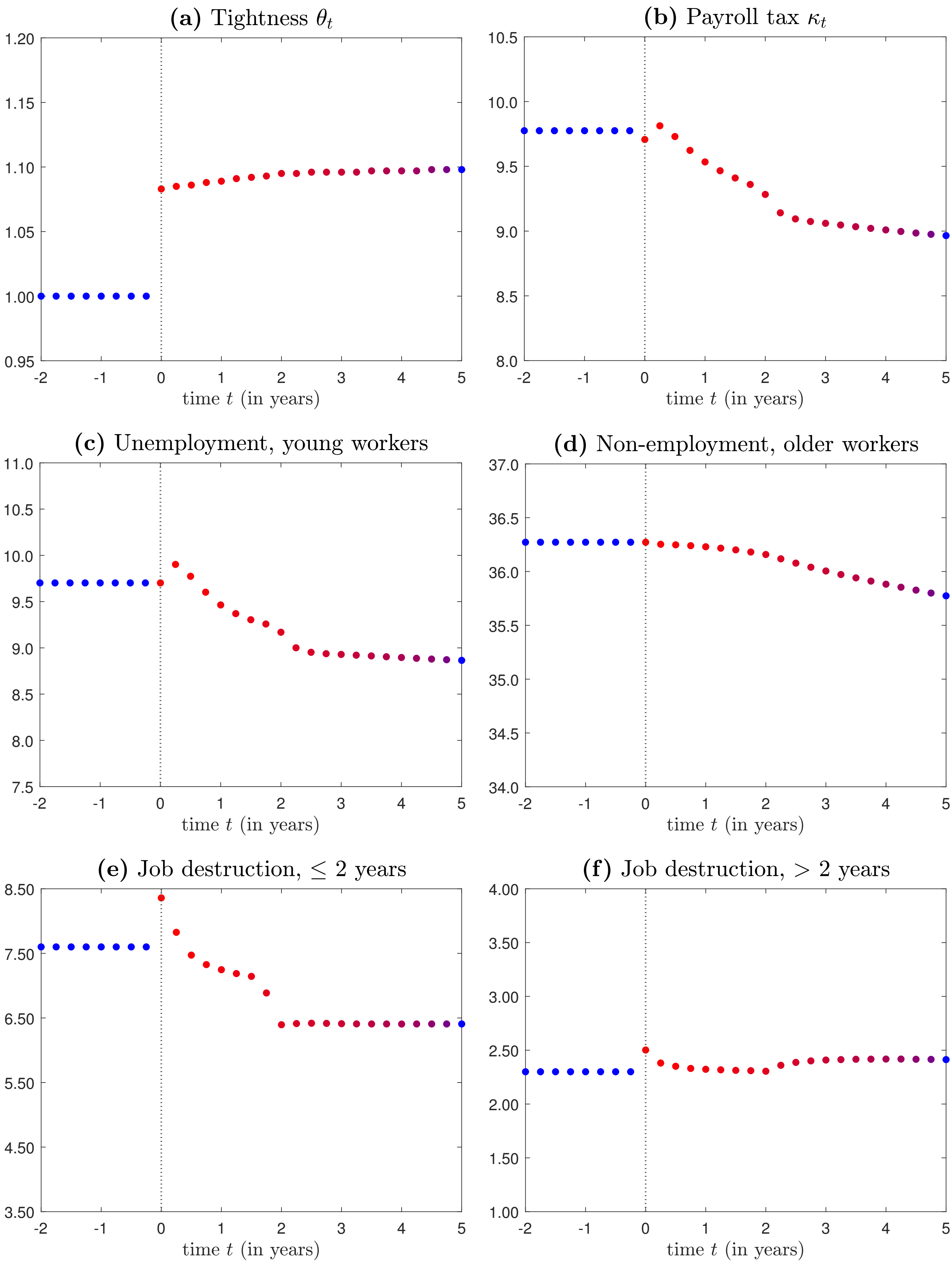

Figure D1. Transition dynamics under a *statu-quo* reform

Notes: The figure displays the time path of several labour market variables during the transition towards unified EPL under a partially non-retroactive reform. Figures on the vertical axis are expressed in percent, except for tightness θ (Figure D1a) which is reported in levels. On the horizontal axis, time is measured in years relative to the introduction of the unified EPL scheme, which occurs in period 0.

E The Approximate Model with Savings

To understand how workers faced with the wages and labour market trajectories of the baseline model would make unrestricted consumption-savings decisions, we use a so-called "approximate model with savings". This model consists of two components: (i) the labour-market part coming from the baseline model which generates the earnings processes (wages and transition probabilities of moving in and out of employment), and (ii) the incomplete-markets part of the model where the earnings processes are taken as exogenous and agents use a risk-free asset to smooth consumption. The next sections are devoted to presenting this model.[1]

E.1 Economic Environment

The labour-market part of the model is almost identical to the baseline model, so we omit a detailed repetition of its equations. There are two modifications. First, workers discount future utilities by a subjective discount rate ϱ (instead of r as in the baseline model) which corresponds to the discount factor used in the incomplete-markets part of the model as well. Second, we assume that per-period consumption for a worker contains an interest income component $r\bar{a}$ as well, where $\bar{a}$ denotes the average asset level in the economy. Recall that the Nash-bargained wage depends on the marginal utility of consumption of the worker, $u'(c)$. Under these assumptions, the wage depends on the aggregate stock of savings in the economy, but not on the individual savings decisions of the worker.[2]

Given parameters and a value for $r\bar{a}$, the labour-market component of the model generates wage functions $w^y(z,\tau)$ and $w^o(z,\tau)$, separation decision rules $\overline{z}^y(\tau)$ and $\overline{z}^o(\tau)$, and a job finding rate $\theta q(\theta)$. These outcomes are used as inputs into the incomplete-markets part of the model which we now turn to describe in more detail.

E.2 Bellman Equations

Since our focus is on stationary equilibria, we omit time indices in order to simplify the notation. We denote by U^i (resp. W^i) the value of being non-employed (resp. being employed), with $i \in \{y, o\}$ indicating the age of the worker. For young workers who are unemployed, the only state variable is the current level of assets of the worker, denoted as a. Thus the value function

[1] We must limit interactions between the labour-market and incomplete-markets parts of the model, since the model that combines fully these two setups is beyond computational reach. With endogenous savings, Nash-bargained wages become a function of the worker's assets, meaning that Nash bargaining becomes a functional fixed-point problem with respect to wages (which we cannot always solve). In addition, the firm's value of a filled job becomes a function of assets, which implies that the free-entry condition depends on the asset distribution of unemployed workers. The problem there is not only that this is an infinite-dimensional object, but also that it eradicates the forward-looking nature of the free-entry condition that enables us to compute transition paths.

[2] This is the only feedback force from the incomplete-markets part to the labour-market part of the model. In contrast, when wages depend on the asset of workers, as in Krusell et al. [2010], this creates an incentive for workers to save in the risk-free asset for the very purpose of bargaining for a higher wage.

U^y solves

$$U^y(a) = \max_{c,a'} \left\{ u(c) + \frac{1}{1+\varrho} \left[(1-\gamma) \left(\theta q(\theta) W^y(z_0, 0, a') + (1-\theta q(\theta)) U^y(a') \right) + \gamma U^o(a') \right] \right\} \tag{1}$$

subject to

$$\begin{aligned} c + a' &\leq b^y + (1+r)a, \\ a' &\geq 0. \end{aligned}$$

As is standard in the literature, we add a retirement phase in order to obtain a realistic savings pattern over the life cycle. Letting R denote the value of being retired, the value function of older non-employed workers, U^o, is given by

$$U^o(a) = \max_{c,a'} \left\{ u(c) + \frac{1}{1+\varrho} \left((1-\chi) U^o(a') + \chi R(a') \right) \right\} \tag{2}$$

subject to

$$\begin{aligned} c + a' &\leq b^o + (1+r)a, \\ a' &\geq 0. \end{aligned}$$

Turning to the value of employment for a young employed worker, her state variables are match productivity z, current job tenure τ, and assets a. A young employed worker solves

$$\begin{aligned} W^y(z,\tau,a) = \max_{c,a'} \Bigg\{ u(c) + \frac{1}{1+\varrho} \Bigg[(1-\gamma) \Bigg(\sum_{z' \geq \overline{z}^y(\tau')} \pi_{z,z'} W^y(z', \tau', a') \\ + \Bigg(1 - \sum_{z' \geq \overline{z}^y(\tau')} \pi_{z,z'} \Bigg) U^y(a' + \phi(\tau')) \Bigg) + \gamma \Bigg(\sum_{z' \geq \overline{z}^o(\tau')} \pi_{z,z'} W^o(z', \tau', a') \\ + \Bigg(1 - \sum_{z' \geq \overline{z}^o(\tau')} \pi_{z,z'} \Bigg) U^o(a' + \phi(\tau')) \Bigg) \Bigg] \Bigg\} \end{aligned} \tag{3}$$

subject to

$$\begin{aligned} c + a' &\leq w^y(z,\tau) + (1+r)a, \\ a' &\geq 0. \end{aligned}$$

As is evident in equation (3), the productivity thresholds $\overline{z}^y(\tau)$ and $\overline{z}^o(\tau)$ determine the continuation values of the worker when she is employed. Similarly, the recursive problem of an older employed worker reads

$$W^{o}(z,\tau,a)=\max_{c,a'}\left\{u(c)+\frac{1}{1+\varrho}\left[(1-\chi)\left(\sum_{z'\geq\overline{z}^{o}(\tau')}\pi_{z,z'}W^{o}(z',\tau',a')\right.\right.\right.$$
$$\left.\left.\left.+\left(1-\sum_{z'\geq\overline{z}^{o}(\tau')}\pi_{z,z'}\right)U^{o}(a'+\phi(\tau'))\right)+\chi R(a')\right]\right\} \tag{4}$$

subject to

$$\begin{aligned} c+a' &\leq w^{o}(z,\tau)+(1+r)a, \\ a' &\geq 0. \end{aligned}$$

Once a worker enters retirement, she receives a retirement benefit b^r each period (we do not introduce taxes to finance the provision of b^r to simplify the comparison to the baseline model) and dies with per-period probability ι. The recursive problem is

$$R(a)=\max_{c,a'}\left\{u(c)+\frac{1-\iota}{1+\varrho}R(a')\right\} \tag{5}$$

subject to

$$\begin{aligned} c+a' &\leq b^{r}+(1+r)a, \\ a' &\geq 0. \end{aligned}$$

Dying retirees are replaced by an equally-large measure of new workers to keep the population measure at a constant unit level. Newborn workers start off their lives in unemployment with zero assets.[3]

E.3 Stationary equilibrium

Let $\lambda^y(z,\tau,a)$, $\lambda^o(z,\tau,a)$ denote the distributions of young and older employed workers; $\mu^y(a)$, $\mu^o(a)$ denote the distributions of young and older non-employed; and $\mu^r(a)$ denote the distribution of retired workers. As is standard in the literature, one can construct transition functions describing how the distributions evolve between periods. These transition functions are generated by the separation decision rules $\overline{z}^y(\tau)$, $\overline{z}^o(\tau)$ and savings decisions rules $\overline{a}^y(z,\tau,a)$, $\overline{a}^o(z,\tau,a)$, $\overline{a}^y(a)$, $\overline{a}^o(a)$, $\overline{a}^r(a)$, and by the laws of motion for the exogenous stochastic processes. A stationary equilibrium is then defined by a list of value functions and policy functions solving the workers' problems (1)–(5), and population distributions $\lambda^y(z,\tau,a)$, $\lambda^o(z,\tau,a)$, $\mu^y(a)$, $\mu^o(a)$, $\mu^r(a)$ that are time-invariant.

[3]We ran experiments where the assets holdings of the dead were redistributed as lump-sum transfers to newborns workers. The results were very similar to those presented here.

E.4 Computation

We implement the following fixed-point algorithm. First, we guess the average asset level in the economy, $\bar{a}$, to solve the labour-market part of the model. We then feed the resulting income processes into the incomplete-markets part of the model and compute its stationary equilibrium. We use $\lambda^y(z,\tau,a)$, $\lambda^o(z,\tau,a)$, $\mu^y(a)$, $\mu^o(a)$, $\mu^r(a)$ to update the value of $\bar{a}$, and we iterate until the difference between initial guess and equilibrium $\bar{a}$ is close enough to zero.

E.5 Calibration and Model Outcomes

We set the expected length of the retirement period to 15 years, which implies $\iota = 1/60$. We keep the interest rate unchanged from the baseline model, meaning its value is 1.01 percent per quarter (4 percent per annum). The retirement benefit b^r and the subjective discount rate ϱ are calibrated internally to match two data moments. We set b^r to be 80 percent of average gross earnings during working age.[4] This yields $b^r = 0.2800$. To select a value for the subjective discount rate, we target a wealth-to-income ratio that is consistent with Spanish data. According to the 2008 Spanish Survey of Household Finances (Banco de España [2011]), the ratio between average wealth and average income among working-age households was 2.3.[5] We find that $\varrho = 0.94$ percent (implying a subjective discount rate of 3.7 percent per annum) delivers this value in the model.

Figure E1 shows the policy function for net savings, $\overline{a}^y(a) - a$ and $\overline{a}^o(a) - a$, for non-employed workers (young workers in Figure E1a, older ones in Figure E1b). As can be seen, workers run down their stock of assets for the purpose of smoothing consumption during spells of joblessness. Asset-poor workers are close to being hand-to-mouth, as their possibilities to draw on savings are limited.

The stationary distribution over assets is displayed in Figure E2. As is typical in this class of models, this distribution is skewed to the right, with many workers (8.8% of them) at or near the borrowing constraint. In the benchmark equilibrium, the Gini coefficient of the distribution of assets is 0.61.

[4] See "Replacement Rates", in the "Pensions at a Glance" report from the OECD [2005].

[5] According to the Survey, the mean net wealth of Spanish households is €226,000 and mean income is €39,700. Housing wealth (primary residence) makes up for 59% of total net wealth. Therefore the mean non-housing wealth of household (which proxies assets that may be liquidated on short notice and with small transaction costs to smooth out shocks) is about 2.3 times the mean of household income.

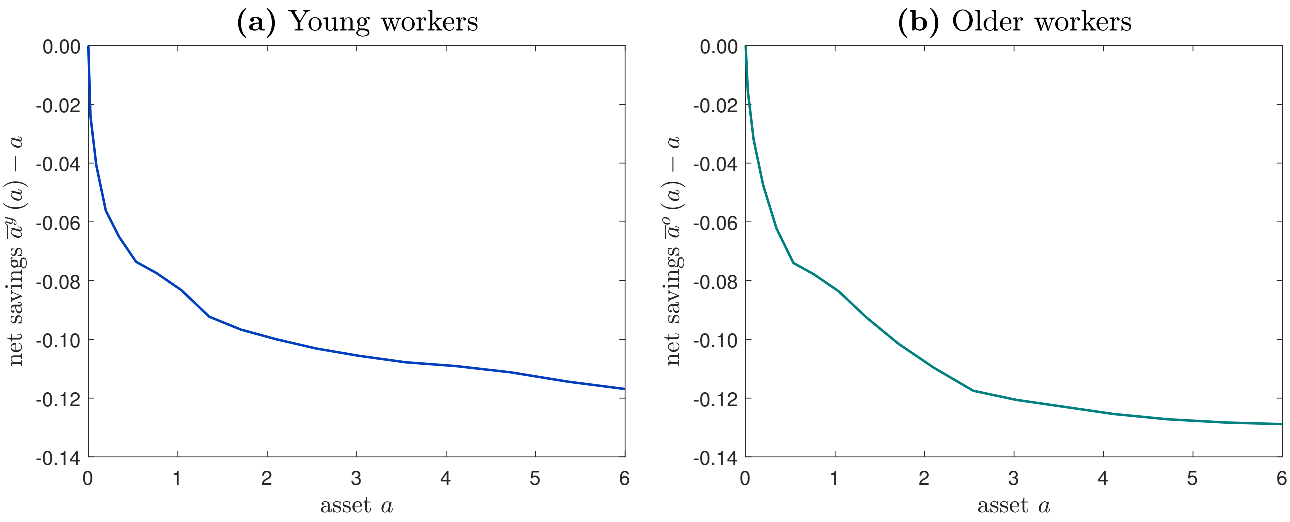

Figure E1. Net savings function of non-employed workers

Notes: The figure shows the net savings function (the policy function on assets in the next period minus asset in the current period) of workers during non-employment. Figure E1a displays the function for young workers while Figure E1b displays the function for older workers.

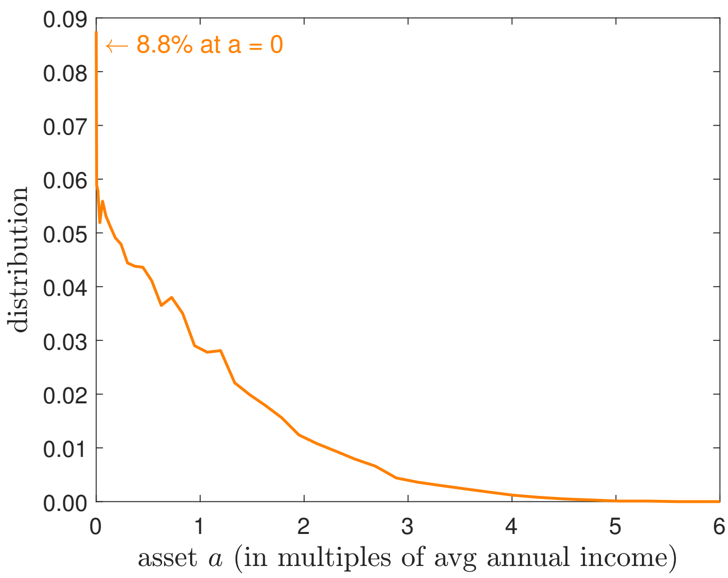

Figure E2. Stationary distribution over assets

Notes: The figure shows the stationary distribution over assets in the approximate model with savings.

F Details of the Model Extensions

F.1 Wage Rigidity

In this version of the model, wages are rigid in the sense that the wage during the current period partly depends on its value in the previous period. Therefore, we need to introduce a new state variable w for ongoing worker-firm matches. Given w, the wage in the current period, which we denote as w^*, is

$$w^* = \alpha^r w + (1 - \alpha^r)\, w_{\text{NB}} \tag{1}$$

where $0 \leq \alpha^r \leq 1$ is the parameter controlling wage rigidity, and w_{NB} denotes the wage that is implied by Nash bargaining (details follow). Notice that while w_{NB} is endogenous, the dynamics of rigid wages is governed by the exogenously-given law of motion from equation (1). The baseline model corresponds to $\alpha^r = 0$.

A key feature of this environment is that job separation decisions are no longer the joint outcome of bargaining between agents, which implies that the worker must compute her continuation value by taking the separation decision of the firm as given, and vice versa. We thus introduce job separation decisions rules $\overline{z}^y_{W,t}(\tau, w)$ and $\overline{z}^o_{W,t}(\tau, w)$ (resp. $\overline{z}^y_{J,t}(\tau, w)$ and $\overline{z}^o_{J,t}(\tau, w)$) for the worker (resp. the firm). As should be evident, they depend not only on job tenure but also on the wage inherited from the previous period.

Bellman Equations. In newly-formed matches, we assume that the wage starts off at $w_0 = w^y_{\text{NB}}(z_0, 0)$. Thus, the value of a young unemployed worker is given by:

$$\begin{aligned} U^y_t(\Delta, \tau) =& u(a^y(\Delta, \tau) + b^y) + \frac{1}{1+r}\Big[(1-\gamma)\big(\theta_t q(\theta_t)\, W^y_{t+1}(z_0, 0, w_0) \\ & + (1 - \theta_t q(\theta_t))\, U^y_{t+1}(\Delta, \tau)\big) + \gamma \widetilde{U}^o_{t+1}(\Delta, \tau)\Big]. \end{aligned} \tag{2}$$

The other values of non-employment, namely $U^o_t(\tau)$ and $\widetilde{U}^o_t(\Delta, \tau)$, are unchanged from the baseline model. Then, for employed workers and firms with a filled job, the state variable of the wage evolves according to: $w' = w^*$, and the Bellman equations governing their behaviour are:

$$
\begin{aligned}
W_t^y(z,\tau,w) = u\left(w^*\right) + \frac{1}{1+r}\Bigg[(1-\gamma)\Bigg(\Bigg(1 - \sum_{z' \geq \bar{z}^y_{J,t+1}(\tau',w')} \pi_{z,z'}\Bigg) U^y_{t+1}\left(\Delta_{t+1},\tau'\right) \\
+ \sum_{z' \geq \bar{z}^y_{J,t+1}(\tau',w')} \pi_{z,z'} \max\left\{W^y_{t+1}(z',\tau',w'), U^y_{t+1}\left(\Delta_{t+1},\tau'\right)\right\}\Bigg) \\
+ \gamma\Bigg(\Bigg(1 - \sum_{z' \geq \bar{z}^o_{J,t+1}(\tau',w')} \pi_{z,z'}\Bigg) U^o_{t+1}\left(\tau'\right) \\
+ \sum_{z' \geq \bar{z}^o_{J,t+1}(\tau',w')} \pi_{z,z'} \max\left\{W^o_{t+1}(z',\tau',w'), U^o_{t+1}\left(\tau'\right)\right\}\Bigg)\Bigg],
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
W_t^o(z,\tau,w) = u\left(w^*\right) + \frac{1-\chi}{1+r}\Bigg(\Bigg(1 - \sum_{z' \geq \bar{z}^o_{J,t+1}(\tau',w')} \pi_{z,z'}\Bigg) U^o_{t+1}\left(\tau'\right) \\
+ \sum_{z' \geq \bar{z}^o_{J,t+1}(\tau',w')} \pi_{z,z'} \max\left\{W^o_{t+1}(z',\tau',w'), U^o_{t+1}\left(\tau'\right)\right\}\Bigg),
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
J_t^y(z,\tau,w) = z - (1+\kappa_t)w^* + \frac{1}{1+r}\Bigg[(1-\gamma)\Bigg(-\Bigg(1 - \sum_{z' \geq \bar{z}^y_{W,t+1}(\tau',w')} \pi_{z,z'}\Bigg) \phi\left(\tau'\right) \\
+ \sum_{z' \geq \bar{z}^y_{W,t+1}(\tau',w')} \pi_{z,z'} \max\left\{J^y_{t+1}(z',\tau',w'), -\phi\left(\tau'\right)\right\}\Bigg) \\
+ \gamma\Bigg(-\Bigg(1 - \sum_{z' \geq \bar{z}^o_{J,t+1}(\tau',w')} \pi_{z,z'}\Bigg) \phi\left(\tau'\right) \\
+ \sum_{z' \geq \bar{z}^o_{W,t+1}(\tau',w')} \pi_{z,z'} \max\left\{J^o_{t+1}(z',\tau',w'), -\phi\left(\tau'\right)\right\}\Bigg)\Bigg],
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
J_t^o(z,\tau,w) = z - (1+\kappa_t)w^* + \frac{1-\chi}{1+r}\Bigg(\Bigg(\sum_{z' \geq \bar{z}^o_{J,t+1}(\tau',w')} \pi_{z,z'} - 1\Bigg) \phi\left(\tau'\right) \\
+ \sum_{z' \geq \bar{z}^o_{W,t+1}(\tau',w')} \pi_{z,z'} \max\left\{J^o_{t+1}(z',\tau',w'), -\phi\left(\tau'\right)\right\}\Bigg).
\end{aligned}
\tag{6}
$$

In addition to computing these asset values, we also calculate $W_t^y(z,\tau)$, $W_t^o(z,\tau)$, $J_t^y(z,\tau)$, $J_t^o(z,\tau)$ through equations (8)–(11) in order to recover Nash-bargained wages, w_{NB}. When computing these, we use the unemployment value $U_t^y(\Delta, \tau)$ from equation (2). The Nash-bargained wages used to set rigid wages are therefore different from the Nash-bargained wages of the baseline model because the outside option of the worker is taken from equation (2) instead of equation (5) (which defines U_t^y in the baseline model).

Free Entry. The free entry condition in period t is:

$$\frac{k}{q(\theta_t)} = \frac{1}{1+r} J_{t+1}^y(z_0, 0, w_0). \tag{7}$$

Stock-flow Equations. In the stock-flow equations of the model, we must take account of the new state variable w and the separation decision rules $\overline{z}_{W,t}^y(\tau, w)$, $\overline{z}_{W,t}^o(\tau, w)$, $\overline{z}_{J,t}^y(\tau, w)$, $\overline{z}_{J,t}^o(\tau, w)$. The distribution evolves between t and $t+1$ according to:

$$\lambda_{t+1}^y(z_0, 0, w_0) = \theta_t q(\theta_t)(1-\gamma)\sum_{\tau} \mu_t^y(\tau), \tag{8}$$

$$\lambda_{t+1}^y(z', \tau', w') = \sum_w \sum_z \mathbb{1}\left\{z' \geq \max\left\{\overline{z}_{W,t+1}^y(\tau', w'), \overline{z}_{J,t+1}^y(\tau', w')\right\}\right\} \pi_{z,z'}(1-\gamma)\lambda_t^y(z, \tau, w), \tag{9}$$

$$\lambda_{t+1}^o(z', \tau', w') = \sum_w \sum_z \mathbb{1}\left\{z' \geq \max\left\{\overline{z}_{W,t+1}^o(\tau', w'), \overline{z}_{J,t+1}^o(\tau', w')\right\}\right\} \pi_{z,z'}\left(\gamma\lambda_{t+1}^y(z, \tau, w) + (1-\chi)\lambda_{t+1}^o(z, \tau, w)\right). \tag{10}$$

As for the pool of non-employed workers, the dynamics of $\mu_t^y(0)$ is unchanged from the baseline model (see equation (A5)) but that of $\mu_t^y(\tau)$ with $\tau > 0$ and $\mu_t^o(\tau)$ changes to:

$$\mu_{t+1}^y(\tau') = (1-\theta_t q(\theta_t))(1-\gamma)\mu_t^y(\tau') + \sum_w \sum_z \mathbb{1}\left\{z' < \max\left\{\overline{z}_{W,t+1}^y(\tau', w'), \overline{z}_{J,t+1}^y(\tau', w')\right\}\right\} \pi_{z,z'}(1-\gamma)\lambda_t^y(z, \tau), \tag{11}$$

$$\mu_{t+1}^o(\tau') = \gamma\mu_t^y(\tau') + (1-\chi)\mu_t^o(\tau') + \sum_w \sum_z \mathbb{1}\left\{z' < \max\left\{\overline{z}_{W,t+1}^o(\tau', w'), \overline{z}_{J,t+1}^o(\tau', w')\right\}\right\} \pi_{z,z'}(\gamma\lambda_t^y(z, \tau, w) + (1-\chi)\lambda_t^o(z, \tau, w)). \tag{12}$$

Calibration and Model Outcomes. It is instructive to study how the calibrated model parameters change with the degree of wage persistence, α^r. To this end, Table F1 describes results with α^r ranging from 0 (the benchmark equilibrium) to 0.90 (which is our focus in Subsection 6.1). Foremost, when wages are rigid, the gap in EPL at $\tau > 8$ has a larger incidence on job separation at short job tenures. Thus z_0 increases to keep job destruction under 2 years at 7.5 percent (our calibration target). Average wages are higher, which implies that b^y and b^o become higher to match the calibration targets for UI replacement rates. Last, the vacancy posting cost becomes higher too as the expected gains from meeting a worker increase with z_0.

Table F1. Parameter values used in the model with wage rigidity

Parameters matching data moments	**Bench.**	$\alpha^r = 0.25$	$\alpha^r = 0.50$	$\alpha^r = 0.75$	$\alpha^r = 0.90$
matching efficiency A	0.4000	0.4000	0.4000	0.4000	0.4000
unemp. income, young workers b^y	0.2203	0.2295	0.2383	0.2462	0.2672
unemp. income, older workers b^o	0.1616	0.1665	0.1717	0.1781	0.1946
vacancy cost k	0.2204	0.2380	0.2540	0.2760	0.3015
exogenous separation probability δ	0.0050	0.0050	0.0050	0.0050	0.0050
initial match prod. z_0	0.2800	0.3000	0.3200	0.3400	0.3900
standard dev. of match prod. shock σ	0.0440	0.0440	0.0440	0.0440	0.0440
Parameters of the EPL scheme	**Bench.**	$\alpha^r = 0.25$	$\alpha^r = 0.50$	$\alpha^r = 0.75$	$\alpha^r = 0.90$
entry phase (in months) τ_u	5	4	2	1	1
tenure profile (in d.w.y.s.) ρ_u	20	21	21	23	23

Notes: The top panel reports calibrated parameter values used in the benchmark equilibrium ('Bench.') and in several versions of the model with wage rigidity indexed by the parameter α^r. The bottom panel reports the characteristics of the unified EPL scheme obtained for each set of parameter values.

F.2 Initial Match Heterogeneity

In this version of the model, there is heterogeneity in match productivity upon meeting. It is assumed that z is drawn initially from a distribution $\pi_{0,z}$ which is a mixture of two distributions: a Normal distribution with mean z_0 and standard deviation σ, and a degenerate distribution localised at z_0. The weight on the Normal distribution is α^i, so that when $\alpha^i = 0$ all job-matches start at the same productivity level z_0, as in the baseline model. Agents observe the initial productivity draw and decide whether to start producing or walk away from one another. This introduces a new economic decision in the model, namely a match formation rule.

These modifications may seem benign at first sight, but they have important and non-trivial consequences for the definition and computation of an equilibrium. First, since the probability of matching conditional on meeting is not always equal to 1, the annuity of young workers needs to be adjusted. For example, an unemployment worker who obtained a larger severance package from her previous job has a longer duration of joblessness as she rejects more initial match draws. Thus, the expected duration of the annuity payment becomes a function of the previous job tenure of the worker. Second, and consequently, the value of holding a vacant job must account for the distribution of unemployed workers across previous job tenures. This follows from workers having heterogeneous reservation threshold for match formation, depending on their annuity. Third, there is a wage bargained over upon meeting, where the outside option of the worker is the value of continued search in unemployed with her current annuity. That is, workers can be compensated for giving up their annuity by bargaining for a higher wage upon meeting.

In this version of the model, the annuity payment received by a young worker is:

$$a^y\left(\Delta\left(\tau\right),\tau\right) = \frac{1}{1-\left(1+r\right)^{-\Delta(\tau)}}\frac{r}{1+r}\phi\left(\tau\right), \tag{13}$$

where $\Delta(\tau)$ is the expected duration of joblessness of a worker with previous job tenure τ. Note that workers with a large severance package will tend to be more picky in terms of accepting new job offers. On the flip side, because the annuity is actuarially fair, a longer expected unemployment spell reduces the size of the annuity which will induce workers to accept more job offers again. The annuity balances these forces, and, in our calculations, we never obtain allocations where certain workers would prefer to remain non-employed forever.

Bellman Equations. As should be evident, the transition path of this economy is beyond computational reach. To make vacancy posting decisions, firms would need to keep track of the distribution of unemployed workers across previous job tenure during the transition (see 'Free Entry' below). This a high-dimensional object, which we cannot include in our calculations. We therefore focus on the steady-state equilibrium. We omit time indices from the notations below.

There are two new asset values to be defined in this model: $W^0(z,\tau)$, the value for a young worker of starting a job at productivity level z when her previous job tenure (which determines her current annuity pay) is τ; and $J^0(z,\tau)$, the firm's value of employing such a worker. These asset values enable us to write the value of a young worker being unemployed as:

$$\begin{aligned}U^y(\Delta(\tau),\tau) =& u(a^y(\Delta(\tau),\tau)+b^y)\\&+\frac{1}{1+r}\left[(1-\gamma)\left(\theta q(\theta)\sum_z \pi_{0,z}\max\left\{W^0(z,\tau),U^y(\Delta(\tau),\tau)\right\}\right.\right.\\&\left.\left.+(1-\theta q(\theta))U^y(\Delta(\tau),\tau)\right)+\gamma\widetilde{U}^o(\Delta(\tau),\tau)\right].\end{aligned} \tag{14}$$

Letting $w^0(z,\tau)$ denote the wage negotiated at entry, the values of employment for workers and firms are:

$$\begin{aligned}W^0(z,\tau) =& u\left(w^0(z,\tau)\right)+\frac{1}{1+r}\left((1-\gamma)\sum_{z'}\pi_{z,z'}\max\left\{W^y(z',1),U^y(\Delta(1),1)\right\}\right.\\&\left.+\gamma\sum_{z'}\pi_{z,z'}\max\left\{W^o(z',1),U^o(1)\right\}\right),\end{aligned} \tag{15}$$

$$\begin{aligned}J^0(z,\tau) =& z-(1+\kappa)w^0(z,\tau)+\frac{1}{1+r}\left((1-\gamma)\sum_{z'}\pi_{z,z'}\max\left\{J^y(z',1),-\phi(1)\right\}\right.\\&\left.+\gamma\sum_{z'}\pi_{z,z'}\max\left\{J^o(z',1),-\phi(1)\right\}\right).\end{aligned} \tag{16}$$

The asset values for all the other states of the economy can be computed using the Bellman equations of the baseline model, namely equations (6)–(11).

Wage Setting. To set the wage upon entry, agents maximise the following Nash product:

$$w^0(z,\tau) = \arg\max_w \left\{ \left(W^0(z,\tau;w) - U^y(\Delta(\tau),\tau) \right)^{\beta} \left(J^0(z,\tau;w) \right)^{1-\beta} \right\}. \tag{17}$$

Again, notice that τ in $w^0(z,\tau)$ refers to job tenure in the previous job of the worker, which spills over into her outside option $U^y(\Delta(\tau),\tau)$ (via the annuity) when she bargains with a new firm.

Free Entry. The free-entry condition of this model with initial match heterogeneity depends on the distribution of unemployed workers across previous job tenures. The free-entry condition reads:

$$\frac{k}{q(\theta)} = \frac{1}{1+r} \sum_z \sum_\tau \pi_{0,z} \max\left\{ J^0(z,\tau), 0 \right\} \frac{\mu^y(\tau)}{u}, \tag{18}$$

where $u = \sum_\tau \mu^y(\tau)$ is the number of job seekers (i.e. young unemployed workers). That is, the returns to meeting a worker depend on $\mu^y(\tau)/u$, the conditional probability of the worker having job tenure τ in her previous job.

Stock-flow Equations. The stock-flow equations of the model are almost unchanged from the baseline model. The only changes relate to the stochastic draw of match productivity upon entry and the match formation decision, which we denote as $\overline{z}^0(\tau)$. Employment at the entry level is given by:

$$\lambda^y(z,0)' = \sum_z \pi_{0,z} \mathbb{1}\left\{ z \geq \overline{z}^0(\tau) \right\} \theta q(\theta)(1-\gamma) \sum_\tau \mu^y(\tau) \tag{19}$$

(note on the left-hand side of the equation that we use a prime ($'$) to denote the one-period ahead value of the distribution). The dynamics of the pool of young unemployed worker is now governed by:

$$\mu^y(0)' = \chi\frac{\gamma}{\chi+\gamma} + (1-\theta q(\theta))(1-\gamma)\mu^y(0) + \sum_z \pi_{0,z} \mathbb{1}\left\{ z < \overline{z}^0(0) \right\} \theta q(\theta)(1-\gamma)\mu^y(0) \tag{20}$$

and, for all $\tau' > 0$,

$$\begin{aligned} \mu^y(\tau')' = (1-\theta q(\theta))(1-\gamma)\mu^y(\tau') &+ \sum_z \pi_{0,z} \mathbb{1}\left\{ z < \overline{z}^0(\tau') \right\} \theta q(\theta)(1-\gamma)\mu^y(\tau') \\ &+ \sum_z \mathbb{1}\left\{ z' < \overline{z}^y_{t+1}(\tau') \right\} \pi_{z,z'}(1-\gamma)\lambda^y_t(z,\tau). \end{aligned} \tag{21}$$

Calibration and Model Outcomes. Table F2 reports the calibrated parameter values when α^i ranges from 0.25 to 0.75. To complement the table, Figure F1 shows the expected duration of joblessness, $\Delta(\tau)$, as a function of the worker's previous job tenure.

As can be seen, raising the probability of a stochastic initial draw increases heterogeneity in the duration of joblessness. In the scenario with $\alpha^i = 0.75$, a worker who has remained

Table F2. Parameter values used in the model with initial match heterogeneity

Parameters matching data moments	**Bench.**	$\alpha^i = 0.25$	$\alpha^i = 0.50$	$\alpha^i = 0.75$
matching efficiency A	0.4000	0.4318	0.4912	0.6445
unemp. income, young workers b^y	0.2203	0.2358	0.2583	0.3038
unemp. income, older workers b^o	0.1616	0.1689	0.1795	0.2015
vacancy cost k	0.2204	0.2342	0.2446	0.2492
exogenous separation probability δ	0.0050	0.0050	0.0050	0.0050
initial match prod. z_0	0.2800	0.3094	0.3500	0.4353
standard dev. of match prod. shock σ	0.0440	0.0440	0.0440	0.0440
Parameters of the EPL scheme	**Bench.**	$\alpha^i = 0.25$	$\alpha^i = 0.50$	$\alpha^i = 0.75$
entry phase (in months) τ_u	5	6	4	8
tenure profile (in d.w.y.s.) ρ_u	20	20	11	6

Notes: The top panel reports calibrated parameter values used in the benchmark equilibrium ('Bench.') and in several versions of the model with initial match heterogeneity indexed by the parameter α^i. The bottom panel reports the characteristics of the unified EPL scheme obtained for each set of parameter values.

employed for 30 years prior to job loss faces an expected duration of joblessness of almost 2 years (8 quarters). By contrast, in the baseline model (corresponding to the special case $\alpha^i = 0$) the expected duration of joblessness is uniform across workers, and its value is 3.3 quarters in the benchmark equilibrium. These outcomes require higher matching efficiency, A, and a higher mean for initial productivity draws, z_0, to keep the quarterly job-finding rate equal to 40 percent (our calibration target). Average wages are higher, which implies that b^y and b^o become higher to match the calibration targets for UI replacement rates. Last, the vacancy posting cost becomes higher too as the expected gains from meeting a worker increase with z_0.

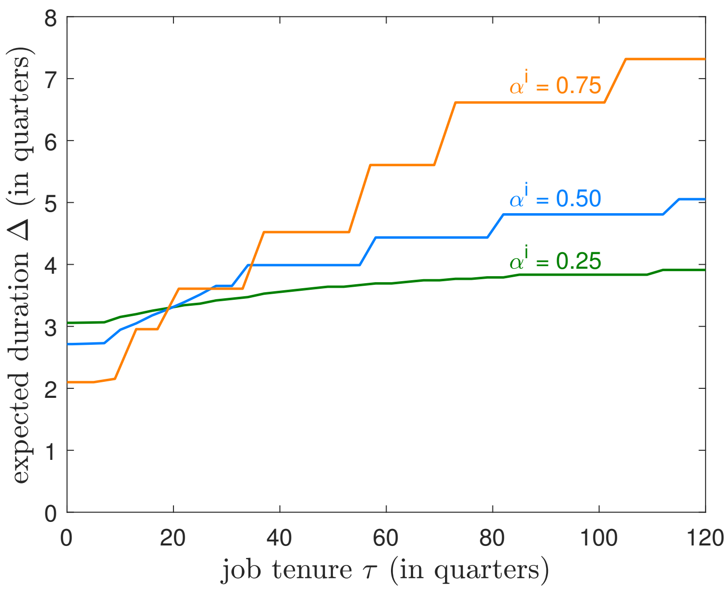

Figure F1. Expected duration of joblessness

Notes: The figure shows the expected duration of joblessness as a function of previous job tenure in three different parameterisations of the model: $\alpha^i = 0.25$, $\alpha^i = 0.50$ and $\alpha^i = 0.75$.

F.3 Human Capital

In this version of the model, the worker-firm pair can devote some effort to acquiring human capital, which increases the flow of output. A job-match with no human capital produces $z\,(1-e)$ units of output, where $0 \leq e \leq 1$ is the (endogenous) effort level, while a job-match with human capital produces $z\left(1+\alpha^h\right)$ units of output. $\alpha^h \geq 0$ is the exogenous parameter controlling the productivity gain from human capital.[6] The probability that effort e delivers human capital accumulation is a concave function $\pi\,(e)$. With probability $1-\pi\,(e)$, effort is unsuccessful and the worker-firm pair must continue to invest in human capital. All jobs start off with no human capital and human capital is firm-specific.[7]

We assume that the worker and the firm bargain over the effort level, e, in addition to bargaining over wages. Thus, we must introduce an additional binary state variable for both agents indicating whether or not the current job-match has acquired human capital.

Bellman Equations. Since newly-formed job-matches start with no human capital, the value of a young unemployed worker is given by:

$$\begin{aligned} U_t^y\,(\Delta,\tau) =& u(a^y\,(\Delta,\tau)+b^y)+\frac{1}{1+r}\Big[(1-\gamma)\left(\theta_t q\,(\theta_t)\,W_{t+1}^{0,y}\,(z_0,0)\right.\\ &\left.+\,(1-\theta_t q\,(\theta_t))\,U_{t+1}^y\,(\Delta,\tau)\right)+\gamma\widetilde{U}_{t+1}^o\,(\Delta,\tau)\Big]. \end{aligned} \tag{22}$$

The other values of non-employment, namely $U_t^o\,(\tau)$ and $\widetilde{U}_t^o\,(\Delta,\tau)$, are unchanged from the baseline model. For employed workers, the asset values are given by:

$$\begin{aligned} W_t^{0,y}(z,\tau) =& u\left(w_t^{0,y}(z,\tau)\right)+\frac{1}{1+r}\Bigg[(1-\gamma)\Bigg(\pi\left(e_t^y(z,\tau)\right)\sum_{z'}\pi_{z,z'}\max\left\{W_{t+1}^{1,y}(z',\tau'),\right.\\ &\left.U_{t+1}^y\,(\Delta_{t+1},\tau')\right\}+\left(1-\pi\left(e_t^y(z,\tau)\right)\right)\sum_{z'}\pi_{z,z'}\max\left\{W_{t+1}^{0,y}(z',\tau'),U_{t+1}^y\,(\Delta_{t+1},\tau')\right\}\Bigg)\\ &+\gamma\Bigg(\pi\left(e_t^y(z,\tau)\right)\sum_{z'}\pi_{z,z'}\max\left\{W_{t+1}^{1,o}(z',\tau'),U_{t+1}^o\,(\tau')\right\}\\ &+\left(1-\pi\left(e_t^y(z,\tau)\right)\right)\sum_{z'}\pi_{z,z'}\max\left\{W_{t+1}^{0,o}(z',\tau'),U_{t+1}^o\,(\tau')\right\}\Bigg)\Bigg], \end{aligned} \tag{23}$$

[6] When $\alpha^h=0$, there are no gains from making any effort – $e=0$ is optimal – so that the economy is identical to that of the baseline model.

[7] Since a job-match always starts with no human capital, human capital cannot be transferred across firms. It is in this sense that human capital is firm- (or job-) specific.

$$W_t^{1,y}(z,\tau) = u\left(w_t^{1,y}(z,\tau)\right) + \frac{1}{1+r}\left((1-\gamma)\sum_{z'}\pi_{z,z'}\max\left\{W_{t+1}^{1,y}(z',\tau'), U_{t+1}^{y}\left(\Delta_{t+1},\tau'\right)\right\}\right.$$
$$\left. +\gamma\sum_{z'}\pi_{z,z'}\max\left\{W_{t+1}^{1,o}(z',\tau'), U_{t+1}^{o}\left(\tau'\right)\right\}\right), \quad (24)$$

$$W_t^{0,o}(z,\tau) = u\left(w_t^{0,o}(z,\tau)\right) + \frac{1-\chi}{1+r}\left(\pi\left(e_t^{o}(z,\tau)\right)\sum_{z'}\pi_{z,z'}\max\left\{W_{t+1}^{1,o}(z',\tau'), U_{t+1}^{o}\left(\tau'\right)\right\}\right.$$
$$\left. + \left(1-\pi\left(e_t^{o}(z,\tau)\right)\right)\sum_{z'}\pi_{z,z'}\max\left\{W_{t+1}^{0,o}(z',\tau'), U_{t+1}^{o}\left(\tau'\right)\right\}\right), \quad (25)$$

$$W_t^{1,o}(z,\tau) = u\left(w_t^{1,o}(z,\tau)\right) + \frac{1-\chi}{1+r}\sum_{z'}\pi_{z,z'}\max\left\{W_{t+1}^{1,o}(z',\tau'), U_{t+1}^{o}\left(\tau'\right)\right\}. \quad (26)$$

Similarly, we write four Bellman equations describing the behaviour of firms in this environment:

$$J_t^{0,y}(z,\tau) = z\left(1-e_t^{y}(z,\tau)\right) - (1+\kappa_t)w_t^{0,y}(z,\tau)$$
$$+ \frac{1}{1+r}\left[(1-\gamma)\left(\pi\left(e_t^{y}(z,\tau)\right)\sum_{z'}\pi_{z,z'}\max\left\{J_{t+1}^{1,y}(z',\tau'), -\phi\left(\tau'\right)\right\}\right.\right.$$
$$\left. + \left(1-\pi\left(e_t^{y}(z,\tau)\right)\right)\sum_{z'}\pi_{z,z'}\max\left\{J_{t+1}^{0,y}(z',\tau'), -\phi\left(\tau'\right)\right\}\right)$$
$$+ \gamma\left(\pi\left(e_t^{y}(z,\tau)\right)\sum_{z'}\pi_{z,z'}\max\left\{J_{t+1}^{1,o}(z',\tau'), -\phi\left(\tau'\right)\right\}\right.$$
$$\left.\left. + \left(1-\pi\left(e_t^{y}(z,\tau)\right)\right)\sum_{z'}\pi_{z,z'}\max\left\{J_{t+1}^{0,y}(z',\tau'), -\phi\left(\tau'\right)\right\}\right)\right], \quad (27)$$

$$J_t^{1,y}(z,\tau) = z\left(1+\alpha^{h}\right) - (1+\kappa_t)w_t^{1,y}(z,\tau) + \frac{1}{1+r}\left((1-\gamma)\sum_{z'}\pi_{z,z'}\max\left\{J_{t+1}^{1,y}(z',\tau'),\right.\right.$$
$$\left.\left. -\phi\left(\tau'\right)\right\} + \gamma\sum_{z'}\pi_{z,z'}\max\left\{J_{t+1}^{1,o}(z',\tau'), -\phi\left(\tau'\right)\right\}\right), \quad (28)$$

$$J_t^{0,o}(z,\tau) = z\left(1-e_t^{o}(z,\tau)\right) - (1+\kappa_t)w_t^{0,o}(z,\tau)$$
$$+ \frac{1-\chi}{1+r}\left(\pi\left(e_t^{o}(z,\tau)\right)\sum_{z'}\pi_{z,z'}\max\left\{J_{t+1}^{1,o}\left(z',\tau'\right), -\phi\left(\tau'\right)\right\}\right.$$
$$\left. + \left(1-\pi\left(e_t^{o}(z,\tau)\right)\right)\sum_{z'}\pi_{z,z'}\max\left\{J_{t+1}^{0,o}\left(z',\tau'\right), -\phi\left(\tau'\right)\right\}\right), \quad (29)$$

$$J_t^{1,o}(z,\tau) = z\left(1+\alpha^h\right) - (1+\kappa_t)w_t^{1,o}(z,\tau) + \frac{1-\chi}{1+r}\sum_{z'}\pi_{z,z'}\max\left\{J_{t+1}^{1,o}\left(z',\tau'\right), -\phi\left(\tau'\right)\right\}. \quad (30)$$

Wage and Effort. In job-matches where human capital has not been acquired yet, workers and firms bargain simultaneously on wages and effort. Thus, these variables solve:

$$\left(w_t^{0,y}\left(z,\tau\right), e_t^y\left(z,\tau\right)\right) = \arg\max_{w,e}\left\{\left(W_t^{0,y}(z,\tau;w,e) - U_t^y\left(\Delta_t,\tau\right)\right)^\beta \times\left(J_t^{0,y}(z,\tau;w,e)+\phi(\tau)\right)^{1-\beta}\right\}, \quad (31)$$

$$\left(w_t^{0,o}\left(z,\tau\right), e_t^o\left(z,\tau\right)\right) = \arg\max_{w,e}\left\{\left(W_t^{0,o}(z,\tau;w,e) - U_t^o\left(\tau\right)\right)^\beta \times\left(J_t^{0,o}(z,\tau;w,e)+\phi(\tau)\right)^{1-\beta}\right\}. \quad (32)$$

On the other hand, after human capital has been acquired, workers and firms bargain on wages only using the same protocol as that in the baseline model (see equations (12) and (13)).

Notice that equations (31) and (32) generate a direct relationship between wages and effort. For instance, for young workers the first-order condition for wages is

$$\beta\frac{u'\left(w_t^{0,y}\left(z,\tau\right)\right)}{W_t^{0,y}(z,\tau) - U_t^y\left(\Delta_t,\tau\right)} = (1-\beta)\frac{1+\kappa_t}{J_t^{0,y}(z,\tau)+\phi(\tau)}, \quad (33)$$

while the first-order condition for effort is

$$\beta\frac{\pi'\left(e_t^y(z,\tau)\right)EW_t^y(z,\tau)}{W_t^{0,y}(z,\tau) - U_t^y\left(\Delta_t,\tau\right)} = (1-\beta)\frac{-z+\pi'\left(e_t^y(z,\tau)\right)EJ_t^y(z,\tau)}{J_t^{0,y}(z,\tau)+\phi(\tau)}. \quad (34)$$

In this equation, $EW_t^y(z,\tau)$ (resp. $EJ_t^y(z,\tau)$) is the expected increase in the worker's (resp. firm's) asset value of employment from acquiring human capital.[8] Combining equations (33) and

[8] We have:

$$EW_t^y(z,\tau) = \frac{1}{1+r}\left[(1-\gamma)\left(\sum_{z'}\pi_{z,z'}\max\left\{W_{t+1}^{1,y}(z',\tau'), U_{t+1}^y\left(\Delta_{t+1},\tau'\right)\right\} - \sum_{z'}\pi_{z,z'}\max\left\{W_{t+1}^{0,y}(z',\tau'), U_{t+1}^y\left(\Delta_{t+1},\tau'\right)\right\}\right) + \gamma\left(\sum_{z'}\pi_{z,z'}\max\left\{W_{t+1}^{1,o}(z',\tau'), U_{t+1}^o\left(\tau'\right)\right\} - \sum_{z'}\pi_{z,z'}\max\left\{W_{t+1}^{0,o}(z',\tau'), U_{t+1}^o\left(\tau'\right)\right\}\right)\right] \quad (35)$$

and

$$EJ_t^y(z,\tau) = \frac{1}{1+r}\left[(1-\gamma)\left(\sum_{z'}\pi_{z,z'}\max\left\{J_{t+1}^{1,y}(z',\tau'), -\phi\left(\tau'\right)\right\} - \sum_{z'}\pi_{z,z'}\max\left\{J_{t+1}^{0,y}(z',\tau'), -\phi\left(\tau'\right)\right\}\right) + \gamma\left(\sum_{z'}\pi_{z,z'}\max\left\{J_{t+1}^{1,o}(z',\tau') - \phi\left(\tau'\right)\right\} - \sum_{z'}\pi_{z,z'}\max\left\{J_{t+1}^{0,o}(z',\tau'), -\phi\left(\tau'\right)\right\}\right)\right]. \quad (36)$$

(34), we obtain:

$$\frac{\pi'\left(e_t^y(z,\tau)\right)EW_t^y(z,\tau)}{u'\left(w_t^{0,y}(z,\tau)\right)}=\frac{-z+\pi'\left(e_t^y(z,\tau)\right)EJ_t^y(z,\tau)}{1+\kappa_t}. \tag{37}$$

The left-hand side is the ratio between the value of a marginal change in effort to that of a marginal change in the wage for the worker. This ratio is equated to the value of a marginal change in effort for the firm divided by that of a marginal change in the wage.

Free Entry. Since newly-formed job-matches start off with no human capital, the free entry condition in period t is:

$$\frac{k}{q(\theta_t)}=\frac{1}{1+r}J_{t+1}^{0,y}(z_0,0). \tag{38}$$

Law of motion. The cross-section distribution of employment evolves between t and $t+1$ according to:

$$\lambda_{t+1}^{0,y}\left(z_0,0\right)=\theta_t q\left(\theta_t\right)\left(1-\gamma\right)\sum_{\tau}\mu_t^y\left(\tau\right), \tag{39}$$

$$\lambda_{t+1}^{0,y}\left(z',\tau'\right)=\sum_{z}\mathbb{1}\left\{z'\geq\overline{z}_{t+1}^{0,y}\left(\tau'\right)\right\}\pi_{z,z'}\left(1-\pi\left(e_t^y(z,\tau)\right)\right)\left(1-\gamma\right)\lambda_t^{0,y}\left(z,\tau\right), \tag{40}$$

$$\begin{aligned}\lambda_{t+1}^{1,y}\left(z',\tau'\right)=\sum_{z}\mathbb{1}\left\{z'\geq\overline{z}_{t+1}^{1,y}\left(\tau'\right)\right\}\pi_{z,z'}\Big[&\pi\left(e_t^y(z,\tau)\right)\left(1-\gamma\right)\lambda_t^{0,y}\left(z,\tau\right)\\&+\left(1-\gamma\right)\lambda_t^{1,y}\left(z,\tau\right)\Big],\end{aligned} \tag{41}$$

$$\begin{aligned}\lambda_{t+1}^{0,o}\left(z',\tau'\right)=\sum_{z}\mathbb{1}\left\{z'\geq\overline{z}_{t+1}^{0,o}\left(\tau'\right)\right\}\pi_{z,z'}\Big[&\left(1-\pi\left(e_t^y(z,\tau)\right)\right)\gamma\lambda_{t+1}^{0,y}\left(z,\tau\right)\\&+\left(1-\pi\left(e_t^o(z,\tau)\right)\right)\left(1-\chi\right)\lambda_{t+1}^{0,o}\left(z,\tau\right)\Big],\end{aligned} \tag{42}$$

$$\begin{aligned}\lambda_{t+1}^{1,o}\left(z',\tau'\right)=\sum_{z}\mathbb{1}\left\{z'\geq\overline{z}_{t+1}^{1,o}\left(\tau'\right)\right\}\pi_{z,z'}\Big[&\pi\left(e_t^y(z,\tau)\right)\gamma\lambda_{t+1}^{0,y}\left(z,\tau\right)+\\&+\gamma\lambda_{t+1}^{1,y}\left(z,\tau\right)+\left(1-\chi\right)\lambda_{t+1}^{1,o}\left(z,\tau\right)\Big].\end{aligned} \tag{43}$$

As for the pool of non-employed workers, the dynamics of $\mu_t^y(0)$ is unchanged from the baseline model (see equation (A5)), but the dynamics of $\mu_t^y(\tau)$ with $\tau > 0$ and that of $\mu_t^o(\tau)$ change to:

$$\begin{aligned}
\mu_{t+1}^{y}(\tau') &= (1-\theta_t q(\theta_t))(1-\gamma)\,\mu_t^y(\tau'). \\
&+ \sum_z \mathbb{1}\left\{z' < \overline{z}_{t+1}^{0,y}(\tau')\right\}\pi_{z,z'}\left(1-\pi\left(e_t^y(z,\tau)\right)\right)(1-\gamma)\,\lambda_t^{0,y}(z,\tau) \\
&+ \sum_z \mathbb{1}\left\{z' < \overline{z}_{t+1}^{1,y}(\tau')\right\}\pi_{z,z'}\Big[\pi\left(e_t^y(z,\tau)\right)(1-\gamma)\,\lambda_t^{0,y}(z,\tau) \\
&\qquad\qquad + (1-\gamma)\,\lambda_t^{1,y}(z,\tau)\Big],
\end{aligned} \tag{44}$$

$$\begin{aligned}
\mu_{t+1}^{o}(\tau') &= \gamma\mu_t^y(\tau') + (1-\chi)\,\mu_t^o(\tau') \\
&+ \sum_z \mathbb{1}\left\{z' < \overline{z}_{t+1}^{0,o}(\tau')\right\}\pi_{z,z'}\Big[\left(1-\pi\left(e_t^y(z,\tau)\right)\right)\gamma\lambda_{t+1}^{0,y}(z,\tau) \\
&\qquad\qquad + \left(1-\pi\left(e_t^o(z,\tau)\right)\right)(1-\chi)\,\lambda_{t+1}^{0,o}(z,\tau)\Big] \\
&+ \sum_z \mathbb{1}\left\{z' < \overline{z}_{t+1}^{1,o}(\tau')\right\}\pi_{z,z'}\Big[\pi\left(e_t^y(z,\tau)\right)\gamma\lambda_{t+1}^{0,y}(z,\tau) + \\
&\qquad\qquad + \gamma\lambda_{t+1}^{1,y}(z,\tau) + (1-\chi)\,\lambda_{t+1}^{1,o}(z,\tau)\Big].
\end{aligned} \tag{45}$$

Calibration and Model Outcomes. Our focus is on $\alpha^h = 0.50$, meaning we assume that firm-specific human capital *per se* can increase productivity by 50 percent. The results are robust to increasing α^h further up to 0.66 and 0.75. To parameterise the model, we use: $\pi(e) = \pi_1 e^{\pi_2}$. Since π_1 and π_2 are intimately related to each other, our approach consists in exploring different values for the curvature π_2 and, for each of them, calibrate the scale π_1 so that half of all job-matches produce using human capital. Table F3 reports the results of this calibration exercise.

Figure F2 displays the probabilities $\pi(e^y(z,\tau))$ and $\pi(e^o(z,\tau))$ to show the underlying policy function for acquiring human capital. Several comments are worth making. First, the effort to acquire human capital does not vary by job tenure – that is to say by the generosity of the severance package associated with job tenure. This is because the effect of job tenure is factored into the wage (see equation (37)). Second, human capital effort displays an inverted U-shape with respect to match productivity. There are two countervailing forces at work. On the one hand, higher match productivity in the current period implies higher match productivity in the subsequent periods, which raises the returns to acquiring human capital. On the other, higher match productivity increases the opportunity cost of making efforts during the current period. Third, effort decreases with age (as shown by the downward shift from Figure F2a to Figure F2b, holding z and τ constant). Older workers face a shorter distance to retirement, which lowers the returns to acquiring human capital. Fourth and last, the levels of the probabilities displayed in Figure F2 are low – less than 1 percent per quarter. Our target of having 50 percent of all job-matches produce using human capital implies a low value for the scale parameter, π_1 (π_1 is set to 0.05 for the computations reported in Figure F2; see Table F3). π_1 being close to zero implies low returns to making any effort. On average across all employed workers, the optimal effort level e is under 0.10.

Table F3. Parameter values used in the model with human capital

Parameters matching data moments	**Bench.**	$\pi_2 = 0.25$	$\pi_2 = 0.50$	$\pi_2 = 0.75$
proba. $\pi(e)$ scale parameter π_1	0.0000	0.0370	0.0500	0.0580
matching efficiency A	0.4000	0.4000	0.4000	0.4000
unemp. income, young workers b^y	0.2203	0.2987	0.2945	0.2902
unemp. income, older workers b^o	0.1616	0.2321	0.2298	0.2272
vacancy cost k	0.2204	0.2445	0.2362	0.2261
exogenous separation probability δ	0.0050	0.0050	0.0050	0.0050
initial match prod. z_0	0.2800	0.3400	0.3500	0.3600
standard dev. of match prod. shock σ	0.0440	0.0440	0.0440	0.0440
Parameters of the EPL scheme	**Bench.**	$\pi_2 = 0.25$	$\pi_2 = 0.50$	$\pi_2 = 0.75$
entry phase (in months) τ_u	5	13	13	8
tenure profile (in d.w.y.s.) ρ_u	20	15	15	13

Notes: The top panel reports calibrated parameter values used in the benchmark equilibrium ('Bench.') and in the model with human capital with $\alpha^h = 0.50$ and different values of the curvature parameter π_2. The bottom panel reports the characteristics of the unified EPL scheme obtained for each set of parameter values.

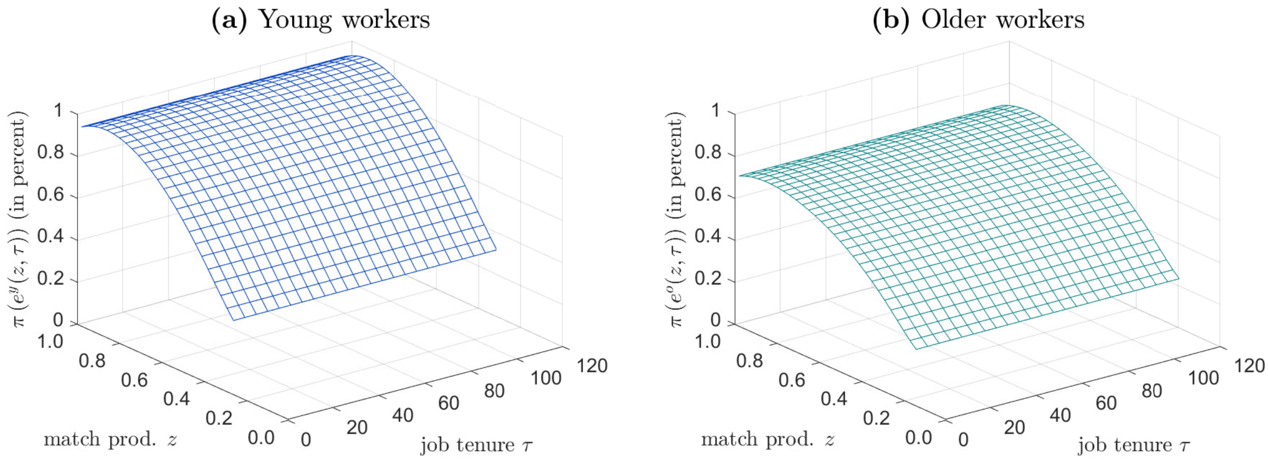

Figure F2. Probability of acquiring human capital

Notes: The figure shows the probability of acquiring human capital as a function of match productivity and job tenure among young (Figure F2a) and older workers (Figure F2b). The model parameters used to construct this figure are: $\alpha^h = 0.50$, $\pi_1 = 0.05$ and $\pi_2 = 0.50$.

G Additional Robustness Checks

Table G1 reports the parameter values used in several alternative calibrations of the model. These alternatives are numbered as follows: (1) the UI replacement rate for young workers is set to 50 percent; (2) the UI replacement rate for young workers is set to 65 percent; (3) the expected duration of the older-age phase (governed by γ) is shortened to 5 years; (4) the expected duration of the older-age phase is raised to 15 years; (5) exogenous separations (viz. job separations triggered by the shock δ) do not entitle the worker to a severance payment; (6) red-tape costs waste half of the total severance package $\phi(\tau)$. In all these parameterisations of the model (as well as in the model extensions studied in Section 6 of the paper), we find that the criterion defining a unified EPL is concave with respect to τ_u and ρ_u.[9] The bottom panel of Table G1 displays the values for τ_u and ρ_u obtained in each calibration.

Table G1. Parameter values used in robustness check exercises

Parameters matching data moments	**Bench.**	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
matching efficiency A	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000
unemp. income, young workers b^y	0.2203	0.1635	0.2803	0.2600	0.1948	0.2370	0.2431
unemp. income, older workers b^o	0.1616	0.1482	0.1753	0.2285	0.1336	0.1862	0.1445
vacancy cost k	0.2204	0.2185	0.2234	0.2280	0.2356	0.2246	0.2624
exogenous separation probability δ	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050
initial match prod. z_0	0.2800	0.2200	0.3400	0.3600	0.2400	0.3100	0.2900
standard dev. of match prod. shock σ	0.0440	0.0440	0.0440	0.0440	0.0440	0.0440	0.0550
Parameters of the EPL scheme	**Bench.**	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
entry phase (in months) τ_u	5	2	8	7	9	6	12
tenure profile (in d.w.y.s.) ρ_u	20	17	24	12	32	16	28

Notes: The top panel reports calibrated parameter values used for the benchmark equilibrium and in sensitivity analyses. The bottom panel reports the parameters of the unified EPL scheme obtained for each set of parameter values. 'Bench.' denotes the benchmark equilibrium; (1) and (2) denote, respectively, lower and higher UI replacement rates for young workers; (3) and (4) denote, respectively, shorter and longer duration of the older age phase; (5) denotes exogenous separation with no severance package; (6) denotes severance packages with red-tape costs.

In Table G2, we report the welfare effects of the transition dynamics in each of the additional robustness check exercises. Robustness checks (1)–(4) are discussed in Subsection 5.3 of the paper, and so we focus on (5) and (6). In scenario (5), where exogenous separations do not entitle workers to severance pay, the minimum service for eligibility barely changes and the slope of the unified EPL scheme decreases only slightly to 16 d.w.y.s. Not surprisingly, the welfare effects shown in Table G2 are very similar to those of the benchmark model. In scenario (6), we consider the effects of adding red-tape costs by assuming that only half of the severance pay, $\phi(\tau)$, is rebated towards the worker (the other half of severance pay is sunk). We find that the entry phase increases to 12 months, and, more importantly, the slope of the unified EPL scheme becomes 28 d.w.y.s. (vs. 20 d.w.y.s. in the benchmark equilibrium). The intuition is that the severance package needs to be made more generous since the share that gets wasted does not help workers to increase consumption during unemployment. The welfare gains of introducing unified EPL, half of which will be lost in red-tape costs, are lower than in the benchmark model (0.46 in Table G2 vs. 1.19 in Table 5).

[9] It seems that, with only two instruments (τ_u and ρ_u) to define the EPL scheme, we reduce the likelihood of having local maxima in the objective function (U^y).

Table G2. Robustness checks: welfare effects of the transition dynamics

(1) Lower UI benefits	**Average**	**1st**	**2nd**	**3rd**	**4th**	**5th**
young workers	1.41	0.69	1.18	1.42	1.78	2.00
older workers	-0.91	-2.36	-1.61	-0.96	-0.02	0.40
(2) Higher UI benefits	**Average**	**1st**	**2nd**	**3rd**	**4th**	**5th**
young workers	0.96	0.52	0.81	0.97	1.15	1.38
older workers	-0.64	-1.78	-1.14	-0.69	0.01	0.37
(3) Shorter older-age phase	**Average**	**1st**	**2nd**	**3rd**	**4th**	**5th**
young workers	2.61	1.94	2.29	2.66	2.94	3.22
older workers	-0.55	-1.87	-1.07	-0.60	0.10	0.68
(4) Longer older-age phase	**Average**	**1st**	**2nd**	**3rd**	**4th**	**5th**
young workers	0.53	0.08	0.40	0.57	0.73	0.91
older workers	-0.23	-0.97	-0.52	-0.18	0.19	0.32
(5) Quits vs. layoffs	**Average**	**1st**	**2nd**	**3rd**	**4th**	**5th**
young workers	1.23	0.64	1.02	1.27	1.46	1.76
older workers	-0.58	-1.65	-1.09	-0.63	0.064	0.40
(6) Red-tape costs	**Average**	**1st**	**2nd**	**3rd**	**4th**	**5th**
young workers	0.46	0.08	0.39	0.49	0.59	0.76
older workers	-0.23	-0.94	-0.51	-0.24	0.15	0.40

Notes: The table reports the welfare changes (measured in consumption-equivalent units) arising from the transition towards unified EPL. 'Average' denotes the cross-sectional average, while '1st', '2nd', '3rd', '4th' and '5th' denote the average within each quintile of the distribution of welfare changes. See text for a description of each panel and Table G1 for calibrated parameter values. All entries are expressed in percent.

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