

Loss of Skill and Labor Market Fluctuations*

Etienne Lalé[†]

University of Bristol

and IZA

Abstract

In this paper, we examine how skill loss can contribute to aggregate labor market fluctuations in the Diamond-Mortensen-Pissarides model. We develop a computationally tractable stochastic version of that model wherein workers accumulate skills on the job and face a risk of skill loss after job destruction. We find that skill heterogeneity dampens the fluctuations of labor market variables, and that introducing skill loss offsets this effect and generates additional amplification. The main forces driving this result are pro-cyclical increases in the probability of skill loss during unemployment: these provide incentives to post proportionally more vacancies during upturns by raising the surplus from employing high-skill workers. Compositional changes in the unemployment pool, on the other hand, play a negligible role for empirically plausible rates of skill depreciation, which imply a relatively slow process compared to the duration of unemployment spells.

Keywords: Diamond-Mortensen-Pissarides model; Labor market volatility; Skill loss

JEL codes: E24; E32; J24; J63; J64.

1 Introduction

In its standard form, the Diamond-Mortensen-Pissarides model (henceforth DMP model) assumes random search, i.e. firms cannot direct their search to specific worker types and vice versa. A direct implication is that productive heterogeneity among unmatched workers matters for job creation. This relationship is in principle mediated by two channels. First, holding the distribution of productive heterogeneity constant over time, job creation should respond to differences in the surplus from employing low-skill vs.

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[†]Address: Department of Economics, University of Bristol, Priory Road Complex, Priory Road, Bristol BS8 1TU, United Kingdom – Phone: +44(0)117 331 7912 – E-mail: etienne.lale@bristol.ac.uk

high-skill workers at different points of the business cycle. Second, holding the surplus from employing each worker type constant, job creation should respond to changes in the distribution of these workers over the business cycle. While these mechanisms are well understood in theory, their quantitative importance remains a topic of active research. The first reason for this is that productive heterogeneity depends on observed as well as unobserved characteristics of workers. As a result, it is difficult to measure its effects on the surplus from employment and how the distribution of worker heterogeneity changes over time. Another important issue is that models in which individual decisions depend on the evolving cross-sectional distribution of the economy are typically difficult to compute.¹ However, ideally one should use a rational expectation model with perfect foresight over this dynamics to assess the implications of productive heterogeneity for job creation.

In this paper, we investigate this issue in the context of a DMP model wherein workers accumulate skills on the job and face a risk of skill loss after job destruction. We focus on skill dynamics as a source of heterogeneity in light of the large literature that studies its impact on unemployment through, e.g., duration dependence due to skill depreciation (Machin and Manning [1999], Layard et al. [2005]) or the loss of specific human capital after job displacement (Pissarides [1992], Rogerson [2005], Wasmer [2006]). Specifically, the framework that we consider is a blend of the DMP model with aggregate shocks and the model put forward by Ljungqvist and Sargent [1998, 2008] to understand the high steady-state unemployment rates triggered by faster skill obsolescence.² We anchor this hybrid model to data on the returns to human capital accumulation and data on labor market transitions prompted by the loss of employment-specific skills. A parsimonious specification of the skill process enables us to compute the stochastic equilibrium while keeping track of the evolving cross-sectional distribution of the economy.

We establish, through a series of numerical experiments, the following results:

- (1) Skill heterogeneity dampens fluctuations in the DMP model. When workers accumulate skills and tend to retain these skills during unemployment, labor productivity is higher and therefore job creation is higher. Thus, on average labor-market tightness resides in the region with less curvature in the matching function, lowering fluctuations in the job-finding rate.
- (2) Cyclical changes in skill loss offset this phenomenon and bring in additional amplification compared to an environment with homogeneous workers. These mechanisms are more pronounced if skills yield a large improvement in worker productivity and/or skills are time-consuming to acquire.
- (3) Gradual skill loss, provided it moves pro-cyclically, increases the surplus from employing more productive workers during upturns. This is the main channel to amplify fluctuations in the model. Compositional changes in the unemployment pool, on the other hand, play a negligible role for empirically plausible values of the probabilities of skill loss: they imply a process of skill depreciation that remains slow relative to the duration of unemployment spells.

¹This is the computational problem usually addressed by means of Krusell and Smith [1998]’s algorithm; see the 2010 special issue of the *Journal of Economic Dynamics and Control* for discussions of this topic.

²Ljungqvist and Sargent [1998, 2008] consider a McCall [1970] job-search model economy with skill accumulation and skill loss. Ljungqvist and Sargent [2007] develop a DMP version of that model. None of these models features aggregate shocks, so that the present paper is, to our best knowledge, the first to construct a stochastic DMP version of that model.

Quantitatively, we find that, compared to an environment with only skill accumulation, the addition of skill loss closes 20 to 45 percent of the distance between the model and data on labor market fluctuations. The amount of amplification depends on the mix between the risk of experiencing a sharp loss of skills, as in the event of job displacement (Jacobson et al. [1993], Davis and von Wachter [2011]), and the risk of losing skills gradually during unemployment (Machin and Manning [1999], Edin and Gustavsson [2008]). The upper bound on the amplification delivered by the model, which we obtain if gradual skill loss is the only mechanism at work, hinges upon a reasonable value of the probability of skill loss: in the extreme scenario where the economy is stuck in the same aggregate state forever, a worker who remains continuously unemployed loses her accumulated skills after ‘only’ 16 months on average. Thus, an empirically plausible process of skill accumulation and skill loss can have substantial implications for the performance of the DMP model via the mechanisms (1)–(3) described above.

It is worth highlighting two features of the analysis to explain the results we obtain for a given set of parameter values.³ First, in the implementation of the model, we consider two skill types (low-skill and high-skill), and we hold the distance between types constant in the experiments comparing, say, the model with only skill accumulation and the model with skill accumulation and skill loss. This magnifies the effects of skill loss because this process undoes completely the increase in productivity resulting from the time-consuming process of skill accumulation. With more skill types, the effects of skill loss on labor market fluctuations would likely be less pronounced.⁴ Second, our model embodies a single matching function lumping together heterogeneous unemployed workers. As noted in the opening paragraph, this feature is standard in the DMP model (e.g. Albrecht [2011]), and is one of our motivations to focus on worker heterogeneity. If, on the other hand, low-skill and high-skill workers were assigned to different matching functions as in Ljungqvist and Sargent [2007] and Gorry et al. [2016], it seems likely that the cyclical response of vacancies would be qualitatively and quantitatively different.

The paper unfolds as follows. The rest of the introduction reviews the related literature. Section 2 presents the model. In Section 3, we select the parameter values of the model and outline our computational strategy. Section 4 contains the main results of the paper. Section 5 concludes. An online appendix provides computational and data details, and the results of several experiments summarized in Section 4.

Related Literature

Numerous studies investigate ways to increase the volatility of labor market variables in the DMP model, following Shimer [2005] and Costain and Reiter [2008]. There are mainly three avenues that are being pursued: changes in the wage setting rule, changes in the model’s calibration, and changes in the model’s specification. This paper falls into the latter category.⁵ We assume wages are set via Nash bargaining throughout the analysis; we establish the results using standard parameter values and calibration targets,

³We thank an anonymous referee for drawing our attention to the issues discussed in this paragraph.

⁴We consider two skill types in order to keep the computational task manageable. The number of state variables in the model increases quadratically with the number of skill types.

⁵Some influential studies that change the wage setting rule of the model include Shimer [2004], Hall [2005], Hall and Milgrom [2008] and Gertler and Trigari [2009]. The paper by Hagedorn and Manovskii [2008] proposes a different calibration strategy of the DMP model that generates labor market fluctuations as large as in the data.

and we show that they are robust to changes in the calibration that are often investigated in the literature.

To relate the findings to studies that consider a DMP model with a richer specification, let us summarize as follows. The model with only skill accumulation and the model with skill accumulation, skill loss and no cyclical changes in skill loss explain only 10 to 15 percent of the volatility of labor market variables. Adding cyclical changes in skill loss increases volatility by a factor ranging between 3 and 6, depending on the main process that leads to skill loss. There are studies exploring channels that yield the same amount of amplification. At the lower end, [Silva and Toledo \[2009\]](#) find that introducing training and separation costs in the DMP model amplifies fluctuations by a factor of 2 to 3; in a New Keynesian version of the model, [Barnichon \[2014\]](#) shows that adding variable labor effort yields an amplification factor of 3. At the other end of this spectrum, [Bils et al. \[2012\]](#) find an increase in the volatility of labor market variables by three quarters when the DMP model is modified to include *ex ante* heterogeneity in productivity and labor supply; [Fujita and Ramey \[2012\]](#) report an amplification factor of 6 in the model with endogenous job separation and no on-the-job search, and they show that on-the-job search closes the remainder of the distance to the data. So, compared to the mechanisms analyzed in these papers, skill loss has similar quantitative properties for improving the cyclical performance of the DMP model.

Within this literature, our analysis has parallels with [Pries \[2008\]](#), [Bils et al. \[2011\]](#) and [Bils et al. \[2012\]](#), who study worker heterogeneity as a means to amplify fluctuations in the stochastic DMP model. [Pries \[2008\]](#) considers *ex ante* heterogeneity in productivity and separation rates. He finds little amplification when the distribution of worker types remains constant, and a strong cyclical response of vacancies when compositional changes are introduced. Shifts in the composition of the pool of searching workers are exogenous in his model. Such shifts play a negligible role in our model because they are endogenous, and we find that these are very limited for empirically plausible values of skill accumulation and skill loss. [Bils et al. \[2011\]](#) report a result similar to ours, that worker heterogeneity can contribute to reducing fluctuations in the DMP model (cf. result (1) above). In their model, this reflects *ex post* heterogeneity in wealth. Our result may seem somewhat more surprising since we analyze productive heterogeneity. [Bils et al. \[2012\]](#) also use a model with differences in productivity as a source of heterogeneity. As already mentioned, they find that it delivers a non-trivial amount of volatility in labor market variables. This finding is due to the fact that productive heterogeneity in their analysis correlates with labor supply preferences, leading to the co-existence of low-surplus and high-surplus workers in the labor market. In our model, on the other hand, differences in the outside option of workers are driven by cyclical changes in the probability of losing accumulated skills. It seems likely that we would obtain larger fluctuations by allowing the flow value of nonemployment to co-move with these changes.

Let us note for completeness that the theme of (*ex ante*) worker heterogeneity is also carefully analyzed in a paper by [Chassamboulli \[2013\]](#). We only briefly mention this study because its focus is on match formation decisions (conditional on meeting) and job separation decisions, which are two margins of the DMP model that we abstract from in the model.

Our analysis is also related to the study by [Gorry et al. \[2016\]](#). The authors analyze *ex post* heterogeneity in a DMP model with worker types that differ with respect to productivity, job finding and job separation rates. They focus on the *persistence* of labor market fluctuations in the DMP model, which is

a complementary issue to the one tackled in this paper. Their approach is also different from ours: they solve for the steady-state equilibrium and then trace the response of unemployment to an unexpected deviation from the equilibrium, whereas we consider a stochastic DMP model with auto-correlated, aggregate productivity shocks.⁶ Interestingly, they find that the so-called thin-market externality does not play a large role in propagating shocks to the model. They report, on the other hand, that compositional changes go a long way increasing the persistence of unemployment fluctuations. Similar to our discussion of the paper by Pries [2008], we think our results complement Gorry et al. [2016] in that we show that large compositional changes are difficult to generate endogenously in the DMP model.

Last, there is a larger literature studying the impact of skill loss on labor market performance under various types of shocks. A seminal paper in this literature is Pissarides [1992]. Pissarides uses a stylized model to show that a one-off increase in the fraction of low-skill workers in unemployment can persist indefinitely through multiple equilibria. Coles and Masters [2000] discuss this phenomenon in relation with labor market policies aimed at training and redeploying workers in long-term unemployment. Ljungqvist and Sargent [1998, 2007, 2008] show that an increase in the rate of skill obsolescence can generate large differences in steady-state unemployment in labor markets with low and high unemployment benefits. Esteban-Pretel and Faraglia [2010] use a New Keynesian model with skill loss during unemployment to study the effects of a negative nominal shock, with the purpose of understanding the trajectory of the Spanish economy before it adopted the euro. Ortego-Martí [2016] develops a DMP model with what he refers to as shocks to ‘unemployment history’, which affect average human capital in the economy. Our paper contributes to this broader literature by studying the relationship between skill loss and aggregate productivity shocks in the rational expectation equilibrium of the stochastic DMP model.

2 The Model

The framework considered is a stochastic Diamond-Mortensen-Pissarides model that we extend to include skill accumulation and skill loss using the construct developed by Ljungqvist and Sargent [1998].

2.1 Environment

Time, indexed by the subscript t , is discrete and runs forever. The economy is populated by a unit continuum of workers and by a continuum of firms. Both types of agents are risk neutral. They use a common rate $r > 0$ to discount the future.

Workers face uncertain working life spans: each period, they are subjected to a probability $\alpha > 0$ of leaving the labor force. A fraction of newborns enters the economy in every period to keep the measure of the labor force at a constant level. Newborn workers are unemployed initially and are endowed with the

⁶It is not so straightforward to compare the quantitative results based on these two approaches. Comparative static results are often used to characterize the cyclical responses of the DMP model without solving for the stochastic equilibrium. The idea is that there is enough persistence in aggregate productivity for the static results to provide a good approximation of the dynamic responses (Mortensen and Nagypál [2007]). The approximation may nonetheless perform poorly when the dynamic system has strong nonlinearities; see Petrosky-Nadeau and Zhang [2016] for a thorough discussion of this issue.

lowest skill level.⁷ Over the course of her working life, a worker accumulates and loses skills according to her own idiosyncratic trajectory. Skills are denoted by x_t , and x_t evolves stochastically over time. There are three Markov chains that govern this process: $p_e(x_t, x_{t+1})$, $p_u(x_t, x_{t+1})$ and $p_d(x_t, x_{t+1})$ denote the transition probability from x_t to x_{t+1} conditional on employment (e), unemployment (u) and job destruction (d), respectively. At the beginning of a period, a worker observes her idiosyncratic skills and the aggregate state of the economy, and she maximizes:

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \left(\frac{1-\alpha}{1+r} \right)^{\tau} \varepsilon_{t+\tau}.$$

\mathbb{E}_t denotes mathematical expectation conditional on information at time t and ε_t is current income. When unemployed, a worker gets an income flow b . b is interpreted as the sum of any nonemployment benefits, home production and the utility of leisure. In employment, a worker receives a wage w_t which is pinned down below by equilibrium conditions.

Firms are infinitely lived. Each firm has at most one job that is either filled or vacant, and a firm can choose to remain inactive. A filled job produces output $z_t x_t$ per period, where z_t is aggregate productivity and x_t is the current skill level of the worker. Workers and firms split the surplus from production by bargaining over the wage w_t . A job is destroyed either by the demographic shock α or, if the worker remains in the labor force, by an exogenous shock that occurs with probability δ . When unmatched, a firm pays a cost $c_v > 0$ to post a vacancy and attract unemployed workers. A free-entry condition holds in every period, so that the expected present discounted value of a vacancy is always zero. Denoting by π_t accounting profits, the objective of firms is to maximize:

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \left(\frac{1}{1+r} \right)^{\tau} \pi_{t+\tau}.$$

Unemployed workers and unfilled vacancies come together via search. Letting u_t denote the number of unemployed and v_t the measure of vacancies created in period t , the number of contacts per unit of time is given by a matching function $m(u_t, v_t)$ with constant returns to scale. After defining labor-market tightness in period t as: $\theta_t \equiv v_t/u_t$, the probability that a randomly-selected unemployed worker finds a vacancy is $f(\theta_t) \equiv m(1, \theta_t)$, and the probability that a randomly-selected vacancy finds a worker is $f(\theta_t)/\theta_t = m(1/\theta_t, 1)$.

Finally, aggregate productivity z_t evolves stochastically over time according to:

$$\ln(z_{t+1}) = \rho_z \ln(z_t) + \sigma_z v_{t+1},$$

where $0 < \rho_z < 1$ and $\sigma_z > 0$. v_t is a standard normally-distributed white noise process. Agents observe the new productivity level at the beginning of the period. Then, since aggregate productivity affects

⁷Under these assumptions, even without skill loss there is a positive fraction of low-skill workers in the labor market. Moreover the distribution of skills always fluctuates since workers who enter the economy during downturns move less rapidly into employment, and therefore they acquire skills later compared to cohorts that entered in good economic times. Obviously, without skill loss the magnitude of these fluctuations is trivial.

the returns to posting a vacancy, v_t and therefore θ_t are also evolving over time. As the duration of unemployment changes, the skill distribution of the overall population changes as well, and agents need to keep track of the distribution in order to compute expectations. We let $\Gamma_t \equiv (\Gamma_{e,t}(x), \Gamma_{u,t}(x))$ denote the cross-sectional distribution at time t , with $\Gamma_{e,t}(x)$ (resp. $\Gamma_{u,t}(x)$) the measure of employed (resp. unemployed) workers with skills x .

2.2 Value Functions

To write the model in recursive form, let W , U and J denote, respectively, the value functions for the employed worker, the unemployed worker and the firm with a filled job. Throughout the analysis, the one-period ahead value of a variable is denoted by a prime ($'$). All value functions depend on aggregate productivity, z , and on the population distribution of workers, Γ .

From the previous section, it follows that the value functions for the worker are:

$$W(x; z, \Gamma) = w(x; z, \Gamma) + \frac{1-\alpha}{1+r} \mathbb{E} \left[\delta \sum_{x'} p_d(x, x') U(x'; z', \Gamma') + (1-\delta) \sum_{x'} p_e(x, x') W(x'; z', \Gamma') \mid z, \Gamma \right], \quad (1)$$

$$U(x; z, \Gamma) = b + \frac{1-\alpha}{1+r} \mathbb{E} \left[\sum_{x'} p_u(x, x') (f(\theta(z, \Gamma)) W(x'; z', \Gamma') + (1-f(\theta(z, \Gamma))) U(x'; z', \Gamma')) \mid z, \Gamma \right]. \quad (2)$$

The value of a filled job for the firm is:

$$J(x; z, \Gamma) = zx - w(x; z, \Gamma) + \frac{1}{1+r} \mathbb{E} \left[(1-\alpha)(1-\delta) \sum_{x'} p_e(x, x') J(x'; z', \Gamma') \mid z, \Gamma \right]. \quad (3)$$

The firm forms expectations about the future skill level of the worker (and future realizations of z and Γ). Notice that equation (3) assumes that the free-entry condition is satisfied.

2.3 Wage Bargaining

As is standard in the DMP model, the wage is the outcome of a Nash bargain between the worker and the firm. Denoting by $\phi \in (0, 1)$ the bargaining power of the worker, we have:

$$w(x; z, \Gamma) = \arg \max \left\{ (W(x; z, \Gamma) - U(x; z, \Gamma))^\phi J(x; z, \Gamma)^{1-\phi} \right\} \quad (4)$$

for all (x, z, Γ) .

2.4 Free Entry

The free-entry condition equates the cost of posting a vacancy to the expected present discounted value of meeting a worker. As discussed previously, this value depends on the future distribution of unemployed workers across skill levels. We thus have:

$$c_v = \frac{1}{1+r} \frac{f(\theta(z, \Gamma))}{\theta(z, \Gamma)} \mathbb{E} \left[\sum_{x'} J(x'; z', \Gamma') \frac{\Gamma'_u(x')}{\sum_x \Gamma'_u(x)} \middle| z, \Gamma \right] \quad (5)$$

where, for instance, $\Gamma'_u(x')/\sum_x \Gamma'_u(x)$ is the probability of meeting an unemployed with skill level x' conditional on meeting a worker at all.

2.5 Law of Motion

Finally, the evolution of the cross-sectional distribution Γ follows from a set of equilibrium flow equations:

$$\Gamma'_e(x') = \sum_x p_u(x, x') f(\theta(z, \Gamma)) (1 - \alpha) \Gamma_u(x) + p_e(x, x') (1 - \delta) (1 - \alpha) \Gamma_e(x) \quad (6)$$

$$\Gamma'_u(x') = \alpha \mathbb{1}_{\{x'=x_\ell\}} + \sum_x p_u(x, x') (1 - f(\theta(z, \Gamma))) (1 - \alpha) \Gamma_u(x) + p_d(x, x') \delta (1 - \alpha) \Gamma_e(x) \quad (7)$$

for all x' , and where x_ℓ is the lower bound on individual skills. Recall that workers are born with the lowest skill level and that they are unemployed initially. A fraction α of them enters the economy in every period, which accounts for $\alpha \mathbb{1}_{\{x'=x_\ell\}}$ in equation (7). In addition, we have: $\sum_x \Gamma_e(x) + \Gamma_u(x) = 1$. Taken jointly, these equations define a mapping T for the distribution Γ , i.e. $\Gamma' = T(\Gamma)$.

2.6 Equilibrium

Having described the environment, value functions and equilibrium conditions, we are in a position to define a stochastic equilibrium. A stochastic equilibrium is a set of value functions $W(x; z, \Gamma)$, $U(x; z, \Gamma)$, $J(x; z, \Gamma)$; a wage $w(x; z, \Gamma)$; labor-market tightness $\theta(z, \Gamma)$; and a law of motion T for the distribution Γ that satisfy four conditions:

- (1) Given θ , w , T , the values W , U , J solve the Bellman equations (1), (2), (3), respectively.
- (2) Given W , U , J , the wage schedule w maximizes the generalized Nash product in equation (4).
- (3) Given J , T , labor-market tightness θ satisfies the free-entry condition (equation (5)).
- (4) Given θ , the law of motion T for the cross-sectional distribution is as described by equations (6) and (7), and the distribution Γ integrates to one.

3 Computation

We specify and calibrate the model in this section. Before we move on to the numerical results, we present the procedure used to evaluate the cyclical properties of the model.

3.1 Specification

To make the computational task manageable, we assume that skills take on two values: $x \in \{x_\ell, x_h\}$. Γ therefore boils down to four numbers (one of which is redundant), so this simplifies the problem of keeping track of the cross-sectional distribution of skills. We refer to a worker with $x = x_\ell$ as a low-skill worker, and a worker with $x = x_h$ as a high-skill worker. We let $x_\ell = 1.0 - \kappa_x$ and $x_h = 1.0 + \kappa_x$ to control the gap between x_ℓ and x_h using a single ‘spread’ parameter, κ_x .

Next, the Markov processes for skill accumulation and skill loss are governed (in somewhat less formal notations) by:

$$p_e(x, x') \sim \begin{bmatrix} 1 - p_e & p_e \\ 0 & 1 \end{bmatrix}; \quad p_u(x, x') \sim \begin{bmatrix} 1 & 0 \\ p_u & 1 - p_u \end{bmatrix}; \quad p_d(x, x') \sim \begin{bmatrix} 1 & 0 \\ p_d & 1 - p_d \end{bmatrix}.$$

p_e is the probability of upgrading skills from x_ℓ to x_h ; p_u is the probability of reverting this process during unemployment; p_d is the probability of losing skills immediately at the time of job loss. As noted in the introduction, p_u captures the idea of gradual skill obsolescence, which is typically linked to duration dependence that lowers the chances of workers to escape long-term unemployment. p_d , on the other hand, captures the idea of job displacement: a worker finds herself with a small probability of regaining employment in a job with the same characteristics as the lost job.

We will also consider below *cyclical changes* in the probabilities of skill loss, p_u and p_d . To do so, we make these probabilities conditional on the aggregate state of the economy, z , by replacing the time-invariant probabilities respectively by:

$$p_u \times (1 + \kappa_u \varphi(z)); \quad p_d \times (1 + \kappa_d \varphi(z)).$$

In these notations, $\varphi(z) \equiv 2 \frac{z-1}{z-\bar{z}}$, and \underline{z} (resp. \bar{z}) denotes the lower (resp. upper) bound on aggregate productivity. Consequently, $\varphi(\underline{z}) = 1$, $\varphi(\bar{z}) = -1$, and $\varphi(1) = 0$ (recall that aggregate productivity fluctuates symmetrically around 1). This construct allows us to introduce cyclical fluctuations in the probabilities of skill loss and control their standard deviations using the parameters κ_u and κ_d .

Finally, we must specify the matching function of the economy. In the baseline experiments, we use the following standard Cobb-Douglas function:

$$m(u_t, v_t) = M u_t^\eta v_t^{1-\eta}.$$

In this specification, M is a parameter measuring matching efficiency and η is the elasticity of the job-filling probability with respect to labor-market tightness.

3.2 Calibration

The number of parameters in the model is sixteen: $r, \rho_z, \sigma_z, \eta, \phi, \delta, \alpha, p_e, p_u, p_d, \kappa_x, \kappa_u, \kappa_d, b, M, c_v$. We choose the values for the first eight parameters using external information. The remaining parameters are calibrated to match several data moments that we discuss below. Throughout the analysis, the model period is set to one week (more precisely: one forty-eighth of a year).

Parameters Set Externally

For $r, \rho_z, \sigma_z, \eta, \phi, \delta$, we use parameter values from the hallmark studies of the cyclical behavior of the DMP model, e.g. [Shimer \[2005\]](#), [Hall \[2005\]](#), [Hall and Milgrom \[2008\]](#), [Hagedorn and Manovskii \[2008\]](#), [Pissarides \[2009\]](#), [Fujita and Ramey \[2012\]](#). The discount rate r is 0.0008 to represent an annualized interest rate of 4 percent. As in [Hagedorn and Manovskii \[2008\]](#) and [Fujita and Ramey \[2012\]](#), the parameters of the aggregate productivity process are $\rho_z = 0.9895$ and $\sigma_z = 0.0034$. We set the elasticity of the job-filling probability with respect to labor-market tightness, η , to 0.50. This is in the range of estimates reported by [Petrongolo and Pissarides \[2001\]](#) and is the value used by [Hall and Milgrom \[2008\]](#) and [Pissarides \[2009\]](#), among others. Following standard practices ([Shimer \[2005\]](#)), we equate the bargaining power of the worker to this value, i.e. we use: $\phi = 0.50$. Finally, the conditional probability of job destruction is $\delta = 0.0050$. The overall job destruction probability is $\alpha + (1 - \alpha)\delta$, so that the monthly separation rate implied by the value of α chosen below is 2 percent.

The other two parameters that we select using external information are α and p_e . We set the probability of leaving the labor market, α , equal to 0.0005 to make the expected length of the working life equal to 40 years. For the probability of upgrading skills, p_e , and later on for $p_u, p_d, \kappa_x, \kappa_u, \kappa_d$, we draw on several results from the literature on human capital. The first of these is that the peak in returns to human capital occurs after a prolonged period of employment in the same job. To lend more precision to this observation, we use the estimates of the returns at 2, 5 and 8 years of tenure reported by [Kambourov and Manovskii \[2009b\]](#). By fitting a quadratic polynomial on their estimates, we find a peak at 14 to 15 years of job tenure.⁸ Thus, we set p_e equal to 0.0014: a worker moves from x_ℓ to x_h on average after 15 years conditional on being employed continuously. We discuss the effects of varying p_e in Subsection 4.2.

Calibrated Parameters

We use standard calibration targets for b, M, c_v . For the flow value of unemployment b , we target a replacement ratio relative to the wage of low-skill workers of 70 percent. $b = 0.70$ is roughly the value proposed by [Hall and Milgrom \[2008\]](#) (see Section II.A of their paper) to account for the flow value of nonemployment in a model with homogeneous workers. It is now the value used in many calibrated versions of the DMP model; see, e.g., [Pissarides \[2009\]](#) and [Fujita and Ramey \[2012\]](#). We calibrate the

⁸In Table 2 (3-digit level for occupations) of [Kambourov and Manovskii \[2009b\]](#), the OLS estimates on tenure at 2, 5 and 8 years are, respectively, 0.0891, 0.1995 and 0.2794. The corresponding numbers based on the IV-GLS estimation are 0.0539, 0.1197 and 0.1680. Let τ indicate tenure. The first set of estimates yields the following profile: $-0.0014 + 0.0487\tau - 0.0017\tau^2$ and the second set yields: $0.0003 + 0.0287\tau - 0.0010\tau^2$. Both profiles imply a maximum value when τ is between 14 and 15.

parameter for matching efficiency, M , to obtain a monthly job-finding rate of 35 percent in line with the long-run behavior of the U.S. labor market. We target a vacancy posting cost, c_v , worth 17 percent of the average of aggregate productivity. This draws on [Fujita and Ramey \[2012\]](#)'s discussion of existing empirical evidence on recruitment costs: they report an investment of 6.7 weekly hours per vacancy filled, amounting to 17 percent of a 40-hour workweek.

For the skill spread parameter, κ_x , we use another finding from the literature on the returns to human capital, that these returns are sizeable. That is, an individual experiences typically a doubling in her wage earnings over the working life which is largely driven by human capital accumulation. As in [Ljungqvist and Sargent \[1998, 2008\]](#), we assume that human capital *per se* can make earnings double. This is in keeping with our focus on the effects of skill loss, and we discuss the effects of changing this assumption in Subsection 4.2 below. Thus, in the benchmark scenario, we calibrate κ_x so as to obtain a high-skill wage twice higher than the wage of low-skill workers.

A crucial part of the calibration is on how to choose targets for the probabilities of skill loss, p_u and p_d . The steps of our reasoning to connect these probabilities to the data are as follows:

- (1) We start by noting that p_u and p_d determine the population distribution of unemployed workers whose skills have deteriorated relative to their skill level in employment. We show in the online appendix that the population measure of these workers in a steady state is:⁹

$$\sum_{x < x_h} \sum_{x' > x} \frac{(1 - f(\theta))(1 - \alpha)p_u(x', x)\Gamma_u(x') + \delta(1 - \alpha)p_d(x', x)\Gamma_e(x')}{1 - (1 - p_u(x, x))(1 - f(\theta))(1 - \alpha)}.$$

When these workers find a job (which occurs with probability $(1 - \alpha)f(\theta)$), they have to acquire skills x again before the wage of their current job matches that of their previous job.

- (2) Next, the literature finds that human capital is to a large extent specific to industries ([Neal \[1995\]](#), [Parent \[2000\]](#)) and occupations ([Kambourov and Manovskii \[2009a,b\]](#)). Therefore we choose to relate the population measure above to the fraction of workers with substantial prior experience in a job, and who change industry/occupation on moving from unemployment to employment. We call this the ‘career-change probability’. In the online appendix, we show how to estimate it from the data.¹⁰ We find that, on average, workers who have been separated from a long-term job and change industry and occupation upon returning to employment account for 4.5 percent of the monthly unemployment-to-employment flows.
- (3) The last remark is that our estimates of the career-change probability yield a calibration target for the *joint* impact of p_u and p_d . Thus, we consider two polar cases. In the first one (the ‘mixed skill loss scenario’), without clear guidance from the data, we assume that p_u and p_d play an equal role

⁹To indicate a steady-state outcome of the model, we omit z and Γ from the notations.

¹⁰We use data from the Current Population Survey (CPS) and from the Job Tenure and Occupational Mobility supplements of the CPS. We estimate two components of the ‘career-change probability’: the probability to change 1-digit industry and occupation upon returning to employment, and the probability that a currently unemployed worker has been employed for at least 10 years in her previous job. Their product yields the so-called career-change probability.

in accounting for the career-change probability.¹¹ In the second scenario, the ‘gradual skill loss scenario’, we posit that $p_u > 0$ and $p_d = 0$. We emphasize it because of the interaction between gradual skill loss and the duration of unemployment, which is an equilibrium outcome of the model.

To summarize, in the model, we look at the share of workers with depleted skills in the unemployment-to-employment flows. In each scenario (i.e. whether p_d is assumed to be non-zero), we calibrate p_u and p_d to make this share replicate the career-change probability measured in the data.

Finally, in the experiments with cyclical deviations in the probabilities of skill loss, we also use the career-change probability to calibrate the parameters κ_u and κ_d . We report in the online appendix that the standard deviation of the cyclical component of the career-change probability is 0.046. Accordingly, in the ‘gradual skill loss scenario’, we calibrate κ_u to make the career-change probability in the model match its empirical counterpart. In the ‘mixed skill loss scenario’, we cut κ_u by half and calibrate κ_d to match the calibration target.¹² Thus, in both scenarios, the standard deviation of the cyclical component of the career-change probability in the model is 0.046. To put this number in perspective, note that it is more than 4 times lower than the standard deviation of cyclical unemployment (Table 2 below).

Outcomes of the Calibration Process

The parameter values are given in Table 1. The bottom panel shows that for b , M , c_v , κ_x , there is no discernible difference between the data moments and the model-generated moments rounded to two decimal places (i.e. the number of decimal places of the targeted moments). It is also worth noting that the values of these parameters are similar in the two scenarios. The reason is that, for empirically plausible values of p_u and p_d , the aggregate quantities of the model (wages and productivity) remain almost unchanged. Next, we see that, according to the calibration, a continuously unemployed worker loses her accumulated skills on average after 2.58 years ($p_u = 0.0108$) or 5.26 years ($p_u = 0.0053$). We find in the data that few unemployed workers change their career after having accumulated substantial experience in an industry or occupation of employment. This suggests that the process governing those career changes is slow compared to the dynamics of unemployment. In the mixed skill loss scenario, we find that 4.25 percent of job separation trigger an instantaneous loss of skills. We think this number is sensible too for the following reasons. First, job displacement is a rare event. Second, the shock prompted by p_d destroys skills that are accumulated only after 15 years, and which are the only source of wage growth in the model. So, we expect to find a small probability for this rather extreme event.

As mentioned previously, the calibration process also delivers values for κ_u and κ_d , which govern cyclical deviations in the probabilities of skill loss. We will discuss these parameters momentarily.

¹¹In our view, this scenario is likely to give an upper bound on the role played by p_d . The reason is that we interpret the shock triggered by p_d as the loss of specific human capital following job displacement. [Davis and von Wachter \[2011\]](#) estimate that job displacements represent 10 to 20% of all job losses, depending on the period considered. Therefore it is unlikely that they explain 50 percent of all the career changes that we find in the data.

¹²Recall that, in the mixed skill loss scenario, we assume a 50:50 split between p_u and p_d in driving the dynamics of the career-change probability. In principle, cutting κ_u by half does not guarantee that gradual skill loss will account for half of the volatility of career changes. We find that in practice it does because there is little interaction between the two sources of skill loss in the numerical experiments.

Table 1. Benchmark parameter values

Parameter	Skill process					
	Mixed skill loss			Gradual skill loss		
r	0.0008			0.0008		
ρ_z	0.9895			0.9895		
σ_z	0.0034			0.0034		
η	0.50			0.50		
ϕ	0.50			0.50		
δ	0.0050			0.0050		
α	0.0005			0.0005		
p_e	0.0014			0.0014		
		Model	Target		Model	Target
b	0.4424	0.7003	0.70	0.4345	0.7003	0.70
M	0.0362	0.3501	0.35	0.0362	0.3499	0.35
c_v	0.1815	0.1699	0.17	0.1814	0.1699	0.17
κ_x	0.3625	1.9995	2.00	0.3862	1.9957	2.00
p_u	0.0053	0.0226	0.0225	0.0108	0.0452	0.0451
p_d	0.0425	0.0225	0.0225	0.0	0.0	0.0

NOTES: r : real interest rate. ρ_z : auto-correlation of aggregate productivity. σ_z : standard deviation of shocks to aggregate productivity. η : elasticity of the job-filling probability with respect to tightness. ϕ : bargaining power of the worker. δ : probability of job destruction. α : probability of leaving the labor market. p_e : probability of upgrading skills. b : flow utility in unemployment. M : aggregate matching efficiency. c_v : vacancy posting cost. κ_x : skill spread between the low-skill and high-skill levels. p_u : probability of losing skills during unemployment. p_d : probability of losing skills upon job destruction.

3.3 Numerical Methodology

Our specification of the skill process alleviates the computational burden: it is possible to keep track of the cross-sectional distribution with only four numbers, and use the constraint on the population size to eliminate one number (see Subsection 3.1 and the online appendix). These numbers correspond to the measure of low-skill and high-skill workers in employment and unemployment. In the computations, we use a grid with 10 points for each population measure of workers. We discretize the stochastic process for aggregate productivity using a Markov chain with 25 states. Finally, during the simulation, we recover $\theta(z, \Gamma)$ by using four-dimensional linear interpolation.

The simulation protocol is as follows. After computing tightness $\theta(z, \Gamma)$ using value function iteration, we simulate the economy for 7,500 periods. We discard the first 4,500 periods and aggregate the remaining observations to quarterly frequency. Thus, we obtain time series that span a period of 62.5 years (again: assuming that a week is one forty-eighth of a year), which is about the observation window of the U.S. time series of productivity and unemployment. The model-generated time series are then logged and detrended using a HP (Hodrick Prescott) filter with smoothing parameter 10^5 . We use these data to compute a set of second moments analyzed in the next section. Finally, the procedure is repeated 1,000 times and the reported results are averages taken over these 1,000 simulations.

4 Results

This section discusses the effects of skill loss on labor market fluctuations in the DMP model under the baseline calibration and in several variants of this calibration.

To provide a framework for the discussion, in Table 2 we report a set of second moments based on U.S. quarterly data before the Great Recession. From left to right, the columns display the standard deviation, the correlation with productivity, the elasticity with respect to productivity and the autocorrelation of unemployment, vacancies and labor-market tightness.¹³ Our focus is on the elasticity with respect to productivity. As highlighted by [Mortensen and Nagypál \[2007\]](#), assessing the performance of the DMP model only on the basis of standard deviations neglects the fact that labor market variables are not perfectly correlated with productivity.¹⁴ To save on space, in the discussion that follows we do not report the correlation between unemployment and vacancies, i.e. the Beveridge relationship; our model with exogenous separations always predicts a negatively-sloped Beveridge curve.¹⁵

Table 2. Business cycle statistics, U.S. quarterly data, 1951Q1–2007Q3

y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$
u_t	0.189	-0.430	-4.009	0.940
v_t	0.192	0.401	3.807	0.944
θ_t	0.371	0.426	7.816	0.945

NOTES: $\sigma(\cdot)$: standard deviation. $\text{Corr}(\cdot, \cdot)$: correlation. u_t : unemployment. v_t : vacancies. θ_t : labor-market tightness. p_t : productivity. The monthly time series are aggregated to quarterly frequency by taking the average of the three monthly values. All time series are taken in log as deviations from a HP trend with smoothing parameter 10^5 .

4.1 Preliminaries

There are two important points worth considering for the numerical experiments. Firstly, which model should be used to determine how skill loss affects the cyclical performance of the DMP model? That is, the framework presented in Section 2 contains three versions of the DMP model on top of the standard model with homogeneous workers (i.e. when $\kappa_x = 0$): (i) a model with only skill accumulation ($p_u = p_d = 0$), (ii) a model with skill accumulation and skill loss ($p_u > 0$ or $p_d > 0$), (iii) a model with skill accumulation, skill loss and cyclical changes in skill loss ($p_u > 0$ and $\kappa_u > 0$, or $p_d > 0$ and $\kappa_d > 0$). Consider first the model with constant rates of skill loss, i.e. model (ii). For empirically plausible values

¹³Unemployment is the Bureau of Labor Statistics data series LNS14000000 (<http://www.bls.gov/data/>); vacancies are the composite Help-Wanted Index computed by [Barnichon \[2010\]](#); productivity is the Federal Reserve Bank of St. Louis data series PRS85006163 (<https://research.stlouisfed.org/fred/>).

¹⁴For the same reason, instead of the value of 20 used in [Shimer \[2005\]](#)'s analysis, [Pissarides \[2009\]](#) retains a value of 7.56 for the elasticity of labor-market tightness with respect to productivity. The latter is very close to the elasticity of 7.82 reported in Table 2.

¹⁵See [Fujita and Ramey \[2012\]](#) and [Chassamboulli \[2013\]](#) for further discussions of this issue. Intuitively, the problem with endogenous separations is that a negative productivity shock generates a higher inflow of newly unemployed workers and thus a lower expected duration to fill a vacancy. This force may lead to a counter-cyclical vacancy rate, and thereby to a positive relationship between unemployment and vacancies.

of p_u and p_d , such as those obtained in Section 3, it is clear that the dynamics of skill loss will remain too slow compared to that of unemployment. Therefore we choose to focus on model (iii) that embodies the richest dynamics through which skill loss can affect labor market fluctuations. By comparing it to model (ii), we measure the effects of deviations of p_u and p_d from their values on average over the business cycle. Later on in the discussion, we also report and analyze the results based on the other two models.

The other remark is on the different role of the types of skill loss captured by $p_u(x, x')$ and $p_d(x, x')$. We highlight here that they differ with respect to the main channel through which they affect the job-creation condition of the model (equation (5)):

$p_u(x, x')$: Its main impact is on the surplus of the firm, i.e. $J(x; z, \Gamma)$. An increase in the risk of losing skills gradually during unemployment lowers the asset value of a high-skill unemployed relative to that of a low-skill unemployed worker. Under Nash bargaining (equation (4)), this increases the surplus from employing a high-skill worker who now has a weaker outside option. On the other hand, if there are no substantial fluctuations in $\theta(z, \Gamma)$ in the first place, then $p_u(x, x')$ has little impact on the composition of the pool of job seekers.

$p_d(x, x')$: Its main impact is on the composition of the pool of job seekers, i.e. $\Gamma_u(x)/\sum_x \Gamma_u(x)$. An increase in the fraction of workers who lose skills immediately on job loss shifts the pool of job seekers towards low-skill workers, which lowers the expected surplus of filling a vacancy. This occurs whether or not fluctuations in $\theta(z, \Gamma)$ are large. On the other hand, $p_d(x, x')$ has little impact on the surplus of the firm: in the asset value of a high-skill worker, the risk of a sharp loss of skills is substantially discounted as per the small probability of job loss δ .

We think this discussion is useful not only for clarifying the mechanisms at work in the model; it also helps understand whether deviations in p_u and p_d should be pro- or counter-cyclical to magnify labor market fluctuations. On the one hand, the probability of losing skills gradually during unemployment should be pro-cyclical so as not to discourage firms from posting more vacancies in good times. On the other hand, the probability of losing skills immediately on job loss should be counter-cyclical to reduce the expected surplus from filling a vacancy in bad times. Both patterns are consistent with the data. The probability of changing career conditional on being unemployed decreases during recessions; see [Carrillo-Tudela et al. \[2014\]](#) and the online appendix. Conversely, the probability of job displacement increases during downturns, as shown by [Davis and von Wachter \[2011\]](#).

4.2 Main Experiments

Having clarified the role of each source of skill loss, we are in a better position to understand their implications when aggregate productivity fluctuates over time.

Table 3 reports the results from the baseline experiments. The table is organized as follows. Panels A and B display the outcomes of the mixed skill loss scenario and of the gradual skill loss scenario, respectively. In each panel, the left part (A1 and B1) presents the results with no cyclical deviations in skill loss (i.e. $\kappa_u = \kappa_d = 0$) while the right part (A2 and B2) allows for cyclicity in the probabilities

Table 3. Labor market fluctuations: Main experiments

Panel A: Mixed skill loss									
A1: No cyclical					A2: Cyclical loss				
y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$	y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$
u_t	0.007	-0.938	-0.380	0.908	u_t	0.019	-0.942	-1.067	0.908
v_t	0.009	0.969	0.516	0.740	v_t	0.025	0.965	1.435	0.736
θ_t	0.015	0.999	0.897	0.851	θ_t	0.043	0.999	2.503	0.850
ζ_t	0.000	-0.712	-0.014	0.946	ζ_t	0.001	0.908	0.074	0.892

Panel B: Gradual skill loss									
B1: No cyclical					B2: Cyclical loss				
y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$	y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$
u_t	0.006	-0.936	-0.306	0.913	u_t	0.033	-0.940	-1.843	0.901
v_t	0.007	0.973	0.413	0.753	v_t	0.044	0.954	2.488	0.708
θ_t	0.012	0.999	0.719	0.860	θ_t	0.073	0.995	4.333	0.835
ζ_t	0.001	-0.728	-0.022	0.946	ζ_t	0.009	0.848	0.477	0.907

NOTES: $\sigma(\cdot)$: standard deviation. $\text{Corr}(\cdot, \cdot)$: correlation. u_t : unemployment. v_t : vacancies. θ_t : labor market tightness. p_t : productivity. ζ_t : fraction of low-skill workers in the unemployment pool. All time series are aggregated to quarterly frequency and taken in log as deviations from a HP trend with smoothing parameter 10^5 . See Subsection 3.3 for details on the simulation protocol. In Panel A2, $\kappa_u = 0.210$ and $\kappa_d = -0.065$; in Panel B2, $\kappa_u = 0.420$ and $\kappa_d = 0.0$.

of skill loss. The results are displayed in a format similar to that of Table 2. To gain additional insights, we also report the cyclical behavior of another model-generated time series, denoted as ζ_t , measuring the share of low-skill workers in the unemployment pool; i.e. $\zeta_t \equiv \Gamma_u(x_t) / \sum_x \Gamma_u(x)$.

There are two main results in Table 3. The first is that the model without cyclical in the probabilities of skill loss exhibits the usual shortcomings of the DMP model, namely too little volatility in labor market variables. On average across Panels A1 and B1, the elasticity of unemployment with respect to productivity is about 11.8 times lower than in the data, that of vacancies is 8.3 times lower and the elasticity of tightness is 9.8 times lower. These results line up well with many studies of the cyclical behavior of the DMP model. For instance, [Shimer \[2005\]](#) argues that the elasticity of labor-market tightness in the DMP model is 20 times lower than in the data. However, this number must be multiplied by the correlation between tightness and productivity ([Mortensen and Nagypál \[2007\]](#), [Pissarides \[2009\]](#)), which is typically around 0.4. So, the performance of the model with no cyclical deviations in skill loss is similar to that of the standard DMP model. We will explain below that this result is largely driven by the fact that skill heterogeneity dampens fluctuations in the model.

The second main finding is that cyclical changes in the probabilities of skill loss increase the volatility of labor market variables substantially. In Panel A2, the elasticities of unemployment, vacancies and tightness are now 3.8, 2.7 and 3.1 times lower than in the data, respectively. The corresponding numbers in Panel B2 are 2.2, 1.5 and 1.8. In other words, cyclical skill loss magnifies fluctuations by a factor ranging from 3 to 6, depending on the relative importance of gradual skill loss. When looking at labor-market tightness, 90 percent of the volatility remains unexplained in the model with no cyclical skill loss (the average of Panels A1 and B1). This figure drops to, respectively, 68 and 45 percent with cyclical

changes in the probabilities of skill loss in Panels A2 and B2. Thus, cyclical skill loss closes up to half of the gap between the model and data.¹⁶

Delving into the mechanisms, we note in Panel B2 of Table 3 that the standard deviation of ζ_t , the share of low-skill workers in the unemployment pool, is only 0.009. This is a relatively small number compared to the other standard deviations reported in the table. Moreover, in line with the discussion in Subsection 4.1, this variable moves pro-cyclically. Compositional changes in the pool of unemployment, therefore, are not the main driving force behind the amplification seen in Panels A2 and B2 of the table. The amplification of fluctuations is instead explained by the fact that, during good times, firms post proportionally more vacancies because they enjoy a higher asset value of employing a high-skill worker. A second observation is that deviations in the probabilities of skill loss do not predict an implausibly fast decay of skills. For instance, in Panel B, the parameter κ_u is 0.420. p_u and κ_u imply that, should the economy remain at the same aggregate productivity level forever, an unemployed worker would lose her accumulated skills on average after 1.36 years (unconditionally). That duration remains large compared to the average duration of an unemployment spell.

We now turn to two issues that provide more insights into the results: (a) what is *per se* the effect of skill heterogeneity on labor market fluctuations? (b) how important are the returns to skills for the mechanisms analyzed in the model?

No Skill Loss

To isolate the effect of skill heterogeneity, Table 4 reports the outcomes of the experiments in the absence of skill loss. Panel 1 is the model with homogeneous workers, i.e. we set the skill spread parameter, κ_x , to 0. Panel 2 is the model with heterogeneity in skills and no skill loss ($p_u = p_d = 0$).

Table 4 gives a more nuanced picture of the results. First, we see in Panel 1 that our parametrization applied to the DMP model with no skill heterogeneity yields a volatility of labor market variables similar to that in Panels A2 and B2 of Table 3. At first sight, this may seem to invalidate the claim that skill loss amplifies labor market fluctuations: relative to the model in Panel 1, the full model presented in Section 2 exhibits the same cyclical properties. However, Panel 2 of the table shows that in the model with skill heterogeneity and no skill loss (which we referred to as model (i) in Subsection 4.1), fluctuations are actually *lower*. When we use that model as a benchmark, we see that it leaves 83 percent of the elasticity of labor-market tightness unexplained. So, relative to model (i), introducing skill loss as is done in Table 3 (main experiments) closes 18 to 46 percent of the distance to the data.

Why does skill heterogeneity dampen labor market fluctuations in the model? The answer to this question follows directly from two basic properties of the DMP model, that labor-market tightness increases with the expected value of filling a vacancy, and that the job-finding probability $f(\theta_t)$ is a concave function of tightness. That is, skill heterogeneity increases the asset value of filling a vacancy since aver-

¹⁶We calculate these numbers as follows. In Panel B1, for instance, the model predicts an elasticity of tightness with respect to productivity of 0.719. Therefore it explains $0.719/7.816 = 9.2$ percent of the volatility. In Panel B2, 55.4 percent of the elasticity of tightness is explained by the model. If we measure the distance between the model and data by the amount of volatility that remains unexplained, then the distance is 90.8 percent in Panel B1 and 44.6 percent in Panel B2.

Table 4. Labor market fluctuations: No skill loss

1: No skill spread ($\kappa_x = 0$)					2: Skill spread ($\kappa_x > 0$) and no skill loss				
y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$	y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$
u_t	0.024	-0.938	-1.300	0.910	u_t	0.010	-0.942	-0.573	0.910
v_t	0.031	0.966	1.775	0.737	v_t	0.014	0.964	0.775	0.733
θ_t	0.053	0.999	3.077	0.851	θ_t	0.023	1.000	1.349	0.850
ζ_t	0.000	0.083	0.001	0.988	ζ_t	0.000	0.073	0.000	0.988

NOTES: $\sigma(\cdot)$: standard deviation. $\text{Corr}(\cdot, \cdot)$: correlation. u_t : unemployment. v_t : vacancies. θ_t : labor market tightness. p_t : productivity. ζ_t : fraction of low-skill workers in the unemployment pool. All time series are aggregated to quarterly frequency and taken in log as deviations from a HP trend with smoothing parameter 10^5 . See Subsection 3.3 for details on the simulation protocol.

age productivity is higher when workers accumulate skills on the job and tend to retain these skills during unemployment.¹⁷ Labor-market tightness is therefore also higher on average under skill heterogeneity. Next, when θ_t lies in the region where there is less curvature in $f(\theta_t)$, the deviations of labor-market tightness from the mean have a lower impact on the job-finding probability, *ceteris paribus*. The result then follows from the key role of the job-finding probability in driving the cyclical behavior of unemployment and vacancies in the DMP model.

Returns to Skills

To understand the role of returns to skills in the model, we repeat the numerical experiments after changing the gap in productivity between x_ℓ and x_h governed by the parameter κ_x . Specifically, we consider deviations by 0.5 above and below the baseline target for the wage-ratio between high-skill and low-skill workers. We also conduct the experiments after changing the expected duration before moving from the low-skill level x_ℓ to the high-skill level x_h . We modify p_e to obtain 50 percent deviations in this duration. To save on space, we defer the complete results of these experiments to the online appendix.

We obtain the following results. First, we confirm the main findings of this subsection, that cyclical changes in skill loss close a sizeable part of the gap between the model and data. In the baseline experiments, they amplify fluctuations by a factor ranging from 3 to 6. In the experiments with either a lower κ_x or a lower $1/p_e$ this factor ranges between 2 and 4–5, while in the experiments with a higher κ_x or a higher $1/p_e$ this factor is between 3 and 6–7. Second, as shown by these numbers, the amount of amplification in the model increases with the returns to skills. It is intuitive that the effects of deviations in p_u and p_d are more potent when they destroy skills that yield a larger productivity gap (high κ_x) or when these are more time-consuming to acquire (high $1/p_e$). Third, if on the other hand skill heterogeneity matters less quantitatively, then this brings the model closer to a world with homogeneous workers and the volatility of labor market variables in the absence of skill loss is higher. This is in line with the analysis of Table 4 in the previous paragraphs. In sum, changes in returns to the skill component, x , lead to consistent changes in the results presented in this section.

¹⁷For instance, in Panel 2 of Table 4 labor productivity is 15 percent higher than in the model with no skill heterogeneity.

4.3 Further Analyses

We have shown that skill heterogeneity dampens fluctuations in the DMP model, and that adding skill loss to this environment yields a substantial improvement in the cyclical properties of the model. These results have been established based on a calibration that could be described as ‘mainstream’: we have borrowed the parameters and calibration targets that are most commonly used in the literature following the studies by [Mortensen and Nagypál \[2007\]](#), [Hall and Milgrom \[2008\]](#), [Pissarides \[2009\]](#) and [Fujita and Ramey \[2012\]](#), among others. In this subsection, we investigate how the results change when we move away from that environment. We consider two influential alternatives for this purpose: a small surplus calibration and a modified free-entry condition. We end this subsection with a discussion about the robustness of the results.

Small Surplus Calibration

The first alternative we consider draws on the calibration proposed by [Hagedorn and Manovskii \[2008\]](#), which is often coined the ‘small surplus calibration’. We change the bargaining power of workers, ϕ , to 0.05 and we target a replacement ratio of 95 percent to calibrate the flow value of unemployment b . We also follow the authors in adopting the matching function proposed by [den Haan et al. \[2000\]](#), namely:

$$m(u_t, v_t) = \frac{u_t v_t}{(u_t^\gamma + v_t^\gamma)^{1/\gamma}}, \quad \gamma > 0.$$

The details and outcomes of the calibration are provided in the online appendix. The results of the numerical experiments are shown in Table 5. Panel A reports the results when there is no skill loss: A1 is based on the model with no skill heterogeneity and A2 on the model with heterogeneity and no skill loss. Panels B and C display the outcomes of the mixed skill loss scenario and of the gradual skill loss scenario, respectively. These two panels are organized in the same way as Table 3.

We first remark on the results displayed in Panel A. As can be seen, in this environment when there is no skill heterogeneity, the elasticity of unemployment is only 1.8 times lower than in the data, that of vacancies is 1.1 times *higher* and the elasticity of tightness is 1.2 times lower than in the data. Thus, in line with the literature, the small surplus calibration is successful in filling a large part of the gap to the data. We also note in Panel A2 that skill heterogeneity deteriorates the performance of the model, so much so that labor market fluctuations are even lower than in the baseline model (cf. Panel A2 of Table 4). This result is due to the mechanism highlighted in the analysis of Table 4, that labor-market tightness under skill heterogeneity tends to lie in the region with less curvature in the job-finding probability. Quantitatively, this effect is very pronounced because the bargaining power of the firm is $1 - \phi = 0.95$, which makes job creation more responsive to the surplus of employing a high-skill worker.

Panels B and C of Table 5 confirm the picture we have been constructing thus far. Firstly, when we compare Panels B1 and C1 to Panel A2, we find that introducing a positive probability of skill loss yields an additional decrease in labor market fluctuations. This is line with the main experiments. Next, we also find that cyclical changes in the probabilities of skill loss revert this process. Meanwhile, in Panels

Table 5. Labor market fluctuations: Small surplus calibration

Panel A: No skill loss									
A1: No skill spread ($\kappa_x = 0$)					A2: Skill spread ($\kappa_x > 0$) and no skill loss				
y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$	y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$
u_t	0.041	-0.939	-2.262	0.908	u_t	0.005	-0.940	-0.249	0.909
v_t	0.074	0.980	4.281	0.773	v_t	0.010	0.987	0.606	0.790
θ_t	0.111	0.998	6.545	0.851	θ_t	0.015	0.999	0.856	0.850
ζ_t	0.000	0.069	0.001	0.988	ζ_t	0.000	0.073	0.000	0.989

Panel B: Mixed skill loss									
B1: No cyclical loss					B2: Cyclical loss				
y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$	y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$
u_t	0.003	-0.938	-0.165	0.912	u_t	0.004	-0.940	-0.236	0.908
v_t	0.007	0.990	0.416	0.801	v_t	0.010	0.988	0.597	0.793
θ_t	0.010	0.999	0.581	0.856	θ_t	0.014	0.999	0.833	0.850
ζ_t	0.000	-0.712	-0.006	0.949	ζ_t	0.000	0.876	0.020	0.903

Panel C: Gradual skill loss									
C1: No cyclical loss					C2: Cyclical loss				
y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$	y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$
u_t	0.002	-0.936	-0.137	0.912	u_t	0.005	-0.958	-0.274	0.889
v_t	0.006	0.991	0.355	0.803	v_t	0.012	0.964	0.694	0.731
θ_t	0.008	0.998	0.492	0.856	θ_t	0.016	0.991	0.969	0.806
ζ_t	0.000	-0.730	-0.010	0.946	ζ_t	0.006	0.817	0.278	0.918

NOTES: $\sigma(\cdot)$: standard deviation. $\text{Corr}(\cdot, \cdot)$: correlation. u_t : unemployment. v_t : vacancies. θ_t : labor market tightness. p_t : productivity. ζ_t : fraction of low-skill workers in the unemployment pool. All time series are aggregated to quarterly frequency and taken in log as deviations from a HP trend with smoothing parameter 10^5 . See Subsection 3.3 for details on the simulation protocol. In Panel B2, $\kappa_u = 0.105$ and $\kappa_d = -0.069$; in Panel C2, $\kappa_u = 0.210$ and $\kappa_d = 0.0$.

B2 and C2, the cyclical performance of the model is mostly similar to that in Panel A2. So, unlike in the main experiments, deviations in p_u and p_d offset the decline in volatility but they do not bring in additional improvements relative to the model with only skill accumulation. This is noteworthy because the DMP model informed by Hagedorn and Manovskii [2008]’s calibration is known to be much more cyclical compared to many alternatives. Thus, this property does not hold when the model features *ex post* work heterogeneity due to skill accumulation and skill loss.¹⁸

Modified Free-entry Condition

We study another alternative to alter the cyclical performance of the DMP model, which is that proposed by Pissarides [2009]: “a simple remodeling of the [matching] costs from proportional to partly fixed and

¹⁸See Hagedorn and Manovskii [2008], Mortensen and Nagypál [2007], and recently Ljungqvist and Sargent [2015] and Petrosky-Nadeau and Zhang [2016] for a discussion of the higher cyclical response of the DMP model under the small surplus calibration. Of course, that result is not supposed to be invariant to the assumptions about heterogeneity among workers or firms (and we do not claim so either).

partly proportional can increase the volatility of tightness and job finding” (Pissarides [2009, p.1341]).¹⁹ Accordingly, we re-write the free-entry condition of the model presented in Section 2 (equation (5)) as:

$$c_v = \frac{1}{1+r} \frac{f(\theta(z, \Gamma))}{\theta(z, \Gamma)} \mathbb{E} \left[\sum_{x'} (J(x'; z', \Gamma') - c_f) \frac{\Gamma'_u(x')}{\sum_x \Gamma'_u(x)} | z, \Gamma \right]. \quad (8)$$

That is, in addition to the per-period cost of a vacancy c_v , there is now a cost c_f that is paid at the time of hiring a worker.²⁰ If \bar{c} denotes the total job-creation cost, then \bar{c} is the sum of this fixed component and of the expected cost which is proportional to the average duration of a vacancy, $\frac{\theta(z, \Gamma)}{f(\theta(z, \Gamma))}$. We calibrate \bar{c} in this environment to match the target assigned to c_v in Section 3, and we assume a 50:50 split between c_v and c_f by letting $c_f = 0.50\bar{c}$.²¹

Panel A1 in Table 6 verifies that adding a fixed component in the total cost of job creation makes the basic DMP model more volatile. The elasticities of unemployment, vacancies and tightness are 2.3, 1.6 and 1.9 times lower than in the data, respectively. These are in the vicinity of the elasticities that we obtain in the baseline experiments *after* introducing cyclical changes in gradual skill loss (cf. Panel B2 of Table 3). Next, when we introduce skill heterogeneity in this environment, we find, again, that it removes a substantial amount of volatility in the behavior of labor market variables: the elasticities are about 40 percent lower in Panel A2 compared to A1. This result is in line with the numerical experiments highlighted in Subsection 4.2 and the numerical experiments with a small surplus calibration discussed previously.

In Panels B and C of Table 6, we show the analogue of the results presented in Table 3. Consider first the mixed skill loss scenario. When we introduce cyclical changes in the probabilities of skill loss, the elasticity of tightness with respect to productivity increases from 1.16 to 3.24. This is an improvement in volatility by a factor of 2.8, which is exactly what we obtain in the baseline experiments. In the gradual skill loss scenario, meanwhile, we see a slightly larger increase in this elasticity: the improvement is by a factor 8.5 (from 0.77 to 6.48) vs. 6.0 in the baseline experiments (from 0.72 to 4.33). This finding dovetails well with the mechanism at work behind the modified free-entry condition. That is, the role of the fixed component c_f is to mitigate the disincentives for pro-cyclical job creation embedded in the DMP model; see, e.g., Pissarides [2009] and the summary provided in Footnote 19. The same idea underlies the pro-cyclicality of the probability of gradual skill loss that is discussed in Subsection 4.1. Therefore these two effects contribute through the same channels to the higher volatility of labor market variables reported in this numerical experiment. This said, the volatility remains of the same order of magnitude as Panel C2 of Table 3.

¹⁹The intuition provided by Pissarides [2009] is that the proportional increase in vacancy costs following an increase in vacancy duration reduces the incentives to create jobs during good times in the standard DMP model. The role of the fixed component of vacancy costs is to make that increase less than proportional. So doing, it avoids discouraging firms to post more vacancies in good times.

²⁰One can think of c_f as the cost of training a newly hired individual prior to beginning work.

²¹Equation (8) generalizes the free-entry condition of the model, so one can think of the model presented in Section 2 as the special case with $c_f = 0$. To our knowledge, there is no well-established evidence on the relative size of the fixed component and proportional component of job-creation costs. We assume a 50:50 split for simplicity. As shown in Panel A1 of Table 6, this is enough to generate a large increase in the volatility of labor market variables.

Table 6. Labor market fluctuations: Modified free-entry condition

Panel A: No skill loss									
A1: No skill spread ($\kappa_x = 0$)					A2: Skill spread ($\kappa_x > 0$) and no skill loss				
y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$	y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$
u_t	0.032	-0.943	-1.736	0.909	u_t	0.014	-0.945	-0.755	0.905
v_t	0.041	0.967	2.320	0.741	v_t	0.018	0.962	1.013	0.726
θ_t	0.069	0.999	4.058	0.853	θ_t	0.030	1.000	1.768	0.845
ζ_t	0.000	0.070	0.001	0.988	ζ_t	0.000	0.071	0.000	0.988

Panel B: Mixed skill loss									
B1: No cyclical loss					B2: Cyclical loss				
y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$	y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$
u_t	0.009	-0.944	-0.501	0.908	u_t	0.025	-0.948	-1.405	0.904
v_t	0.012	0.972	0.661	0.749	v_t	0.032	0.966	1.837	0.735
θ_t	0.020	0.999	1.163	0.856	θ_t	0.055	0.999	3.243	0.849
ζ_t	0.000	-0.727	-0.018	0.946	ζ_t	0.001	0.934	0.055	0.861

Panel C: Gradual skill loss									
C1: No cyclical loss					C2: Cyclical loss				
y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$	y_t	$\sigma(y_t)$	$\text{Corr}(y_t, p_t)$	$\text{Corr}(y_t, p_t) \frac{\sigma(y_t)}{\sigma(p_t)}$	$\text{Corr}(y_t, y_{t-1})$
u_t	0.006	-0.928	-0.331	0.908	u_t	0.050	-0.939	-2.824	0.896
v_t	0.008	0.959	0.435	0.753	v_t	0.064	0.952	3.650	0.720
θ_t	0.013	0.984	0.766	0.858	θ_t	0.110	0.987	6.476	0.840
ζ_t	0.001	-0.733	-0.024	0.942	ζ_t	0.007	0.876	0.391	0.891

NOTES: $\sigma(\cdot)$: standard deviation. $\text{Corr}(\cdot, \cdot)$: correlation. u_t : unemployment. v_t : vacancies. θ_t : labor market tightness. p_t : productivity. ζ_t : fraction of low-skill workers in the unemployment pool. All time series are aggregated to quarterly frequency and taken in log as deviations from a HP trend with smoothing parameter 10^5 . See Subsection 3.3 for details on the simulation protocol. In Panel B2, $\kappa_u = 0.210$ and $\kappa_d = -0.061$; in Panel C2, $\kappa_u = 0.421$ and $\kappa_d = 0.0$.

Robustness of the Results

The small surplus calibration and the modified free-entry condition are two ways of improving the cyclical performance of the DMP model. When we implement these approaches, we find that our main results remain mostly unchanged. The online appendix reports the outcomes from another set of numerical experiments in which we feed the model with more volatile shocks by using a higher standard deviation σ_z in the aggregate productivity process. This leads to virtually the same results. Finally, we obtain a similar picture when we change the returns to skills as in Subsection 4.2 in each of these environments. Thus, in our view, all three sets of experiments support the following conclusion: the amplification generated by skill accumulation and skill loss does not depend on the amount of fluctuations contained in the baseline model with no skill heterogeneity.

We note one caveat to this discussion. The literature to date has identified numerous other alternatives that allow to amplify labor market fluctuations in the DMP model. Of course, it is beyond the scope of this paper to investigate the effects of skill loss in any variant of the DMP model. Accordingly, we should emphasize that our conclusions are robust to alternatives that change the calibration of the model, but

that they may not extend to those variants that modify the main backbone assumptions of the model. We think that some of these would be relevant as well to study the cyclical performance of the model when one allows for skill accumulation and skill loss. To take two examples, wage stickiness (Shimer [2004], Hall [2005], Gertler and Trigari [2009]) and worker search efforts (Gomme and Lkhagvasuren [2015]) are two departures from the standard model which have been put forward to increase the volatility of labor market variables. It is conceivable that they could interact with skill losses in ways that change the volatility of labor market variables. With wage stickiness, the surplus from employing a high-skill worker may be less responsive to changes in her outside option triggered by the increase in the probability p_u during upturns. With search efforts, workers with a lower surplus from employment may choose a lower probability of contacting an employer, thereby mitigating further the effects of compositional changes in the pool of job seekers. Analyzing these issues in depth seems an important endeavor for future research.

5 Conclusion

In this paper, we investigated how skill loss can contribute to aggregate labor market fluctuations in the Diamond-Mortensen-Pissarides model. We found that empirically plausible increases in the probability of skill loss in good times deliver a non-trivial increase in volatility: these changes raise the surplus from employing highly productive workers during upturns, and thereby they make job creation respond more strongly to a positive aggregate shock. On the other hand, we found that compositional changes in the unemployment pool are too limited quantitatively to have a sizeable impact on the cyclical behavior of job creation. These results provide guidance on the cyclical behavior of the Diamond-Mortensen-Pissarides model with productive heterogeneity among workers.

There are other types of heterogeneity that could be considered to study the properties of the model. First, while we have emphasized *ex post* heterogeneity in productivity coming from labor market experience, workers are also heterogeneous *ex ante* in terms of the skills that they bring in the labor market. Second, workers are heterogeneous with respect to other dimensions that can affect job creation. For instance, wealth can change the bargaining position of workers via its effect on the marginal utility of consumption (Bils et al. [2011]). Productive heterogeneity, moreover, can be correlated with other types of worker heterogeneity that matter for job creation as in Bils et al. [2012]. Third and finally, firms are also heterogeneous. Lise and Robin [2016] show that this feature can interact with worker heterogeneity since specific firms can be in demand of certain worker types. Studying the effects of skill loss in some of these settings would be an interesting avenue for future work.

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