Search and Multiple Jobholding[*]

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March 2024

Abstract

This paper develops an equilibrium model of the labor market with hours worked, off- and on-the-job search, and single as well as multiple jobholders. The model quantitatively accounts for the incidence of and worker flows in and out of multiple jobholding. Central to the model's mechanism is that holding a second job ties the worker to her primary employer, providing the benefits of a stronger outside option when bargaining with the outside employer. The model is also informative of how multiple jobholding shapes outcomes that are typically the focus of search models. Multiple jobholding has opposing effects on job-to-job transitions, which mostly offset each other. At the same time, since the option of having second jobs extends the survival of a worker's main job, it reduces job separations and increases the employment rate. These findings have significant implications for calibrating standard models that ignore multiple jobholding.

JEL codes: E24, J21, J62

Keywords: Multiple jobholding; Employment; Hours worked; Job search

[*]The empirical material presented here appeared in IZA Discussion paper no. 10355 [2016] titled "The evolution of multiple jobholding in the U.S. labor market: The complete picture of gross worker flows", which this paper extends and supersedes. I am grateful for constructive comments and suggestions by Björn Brügemann, Domenico Ferraro, François Langot, Hamish Low, Hélène Turon and Erin Wolcott, and participants of the annual Search and Matching Conference at BI Norwegian Business School, the "Economics of Alternative Work Arrangements" workshop at the University of Oxford, the North American Summer Meeting of the Econometric Society, as well as seminar participants at CIRANO, Carleton University, Université du Mans, UQÀM, Arizona State University, Drexel University, and the University of Ottawa. I gratefully acknowledge financial support from Fonds de Recherche du Québec – Société et Culture (Grant number 2019-NP-254270) and the W.E. Upjohn Institute for Employment Research (2018 Early Career Research Grant). All errors are my own.

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1 Introduction

The McCall [1970] model, which lays the foundations for the Diamond [1982]-Mortensen [1982]-Pissarides [1985] model, considers the decision problem of an unemployed worker who searches for jobs, randomly receives offers, and rejects them until she finds a suitable job offer. In a large class of models (e.g., Burdett and Mortensen [1998], Postel-Vinay and Robin [2002], Menzio and Shi [2011]), the worker, once in employment, continues to search for jobs and receives offers from outside employers, and rejects them until an offer prompts her to switch employers. A key restriction in these models (and in the search literature in general) is that the worker cannot accept a job at the outside employer while continuing to work at the incumbent employer – that is, she cannot become a multiple jobholder. As a result, nothing is known about the implications of multiple jobholding for the equilibrium properties of these models and the inferences they provide regarding labor market dynamics.

In this paper, we develop an equilibrium model of the labor market with single as well as multiple jobholders that addresses these questions. Our motivation is twofold. First, multiple jobholding is a quantitatively important phenomenon, with about one in five workers working two jobs simultaneously at some point over a 1-year horizon (see Paxson and Sicherman [1996] and Section 4 of this paper). Second, on-the-job search is pervasive (Fallick and Fleischman [2004]; Faberman et al. [2022]), which implies that many workers find themselves contemplating the option to work two jobs simultaneously at some point. In fact, workers who hold only one job have a higher chance of transitioning into multiple jobholding than of making a job-to-job transition. This simple observation in turn raises an important question: What are the factors that push employed workers to switch jobs instead of combining two jobs at the same time? Standard search models have nothing to say about this trade-off.

From a modeling perspective, the main challenge facing our analysis is how to deal with the repeated interaction between one worker and several (two in this paper) employers. This issue is absent from models à la Burdett and Mortensen [1998] since a worker receiving an outside wage offer that exceeds the current wage moves to the new firm right away. Likewise, in models in the vein of Postel-Vinay and Robin [2002], if the worker holds onto the incumbent employer upon receiving an on-the-job offer, her wage jumps up and she immediately loses contact with the other employer.[1] In contrast, the model developed in this paper explicitly allows the worker to be in contact with two employers for possibly many periods. To accomplish this we need a few key assumptions, which we derive partly from an empirical understanding of how holding a second job affects one's position in the labor market.

We analyze data from the Current Population Survey on transitions in and out of multiple jobholding to gain this understanding. We document two new facts. First, upon taking a second job, and even more so upon returning to single jobholding, the vast majority of workers remain

[1] This 'no-repeated-interaction' logic is not specific to wage posting or sequential auction models. For instance, in Dey and Flinn [2005] and Cahuc et al. [2006], a worker is at some point in contact with two firms and bargains on the wage, but the sequence is played instantaneously until one of the employers can no longer bid up, i.e., the one-worker-two-employers interaction lasts for an infinitesimal portion of time.

at the same primary employer. That is, workers almost never use multiple jobholding as a bridge to transition to a new primary job. Second, holding a second job has no statistically discernible impact on the wages and hours worked of the primary job. This striking fact is consistent with the observation that almost no worker reports a change in the duties or activities of the primary job upon taking or giving up a second job.

Motivated by these observations, we formulate a model in which a worker receiving an on-the-job offer can either reject it to stay with the incumbent employer, move to the new employer, or combine the new job with her current job. In the latter event, she cannot quit to the outside employer at a later date – she turned down that option by becoming a multiple jobholder – unless her older job gets hit by a shock that dissolves the match. Until the worker gives up the second job, she cannot search for another one, though she may do so at any time. In exchange for this commitment, the worker uses the primary job as her outside option to extract a higher surplus from bargaining with the outside (called 'secondary') employer – subject to a participation constraint on the employer's side. These assumptions create an asymmetry between primary and secondary employers which brings two substantial benefits.[2] The first one is tractability. Second, the assumptions put discipline on worker flows in and out of second jobs, allowing us to assess their relevance by comparing these flows to the data.

The other important feature of the model is hours worked. Workers and firms bargain over both wages and hours, which implies that hours are idiosyncratic to the job match(es) between a worker and her employer(s).[3] Because of the asymmetry between jobs, the feasibility of having a second job largely hinges on the working hours dedicated to the primary job: the worker cannot rebargain these hours to free up some time for the second job. To make additional connections to the data, we propose a mapping of hours worked onto labor market services that creates a meaningful distinction between full-time and part-time employment. This construct is relevant not only to analyze multiple jobholding inflows and outflows but also more broadly as it offers a simple solution to capture certain patterns of the intensive margin of labor adjustments (hours per worker) documented in Borowczyk-Martins and Lalé [2019]. Given that these patterns cannot be explained by the recent vintage of search models with fluctuations in hours worked (e.g., Bils et al. [2012], Kudoh and Sasaki [2011], Kudoh et al. [2019], Dossche et al. [2019]),[4] this new construct could be a valuable addition to this class of models.

In sum, the theoretical framework combines a Mortensen and Pissarides [1994]-like model with a structure of very rich adjustments along the intensive margin: in addition to flows in and out of employment, the model features worker movements within the distribution of

[2]In some ways, our model configures an environment that is the polar opposite of the Postel-Vinay and Robin [2002, 2004] world, where the worker would use the outside employer to improve working conditions at the incumbent employer. Unlike Postel-Vinay and Robin [2002, 2004], our model does not aim to describe a job ladder. Its aim is to capture spells of second job holding that bring extra income to the worker and typically last for a short period of time, possibly due to binding participation constraints.

[3]Since hours vary at the job-match level, hours worked change even among *job stayers*. Empirically, these changes are lower than for job changers, but they remain substantial (Borowczyk-Martins and Lalé [2019]).

[4]Borowczyk-Martins and Lalé [2019] show that a large share of cyclical adjustments in hours per worker reflect (within-firm) transitions between full-time and part-time employment, and that they generate sizable and lumpy adjustments in individuals' working hours. Search models with hours fluctuations cannot generate these patterns because they typically feature hours that are the outcome of a smooth optimization problem.

hours worked, across employers, as well as movements in and out of multiple jobholding. All these variables are determined endogenously. The key notions related to multiple jobholding, such as the primary and secondary jobs, are also endogenous. And since the model is general equilibrium, it can be used for counterfactual analysis and inference.

Our analysis yields four main results. First, the model performs well at matching the employment share of multiple jobholders and some salient empirical features of weekly working hours. Moreover, it closely aligns with the data regarding worker flows in and out of second jobs. Given that none of these worker flows are targeted by the calibration, we view these results as validation of the model. The model (calibrated to operate at the monthly frequency) is also consistent with several key observations from annual labor market data. It predicts that about 20 percent of all workers work two jobs simultaneously at some point over a 1-year horizon. Comparisons with annual data are useful to reveal the role of underlying heterogeneity in transition rates in and out of multiple jobs generated in equilibrium. They suggest that the model also performs well along this dimension.

The second main set of results concerns the factors that push workers into or pull then from second jobs. The model attributes a key role to two parameters in this respect. The first one is relative on-the-job search intensity – the rate at which employed job seekers receive offers compared with nonemployed job seekers. Perhaps counterintuitively, search intensity has an ambiguous effect on multiple jobholding. A lower on-the-job search intensity reduces access to second jobs, but it induces multiple jobholders to hold on longer to their second job, given that these jobs become harder to come by. The other key parameter is the flow cost of working a second job, which is additional to the flow cost of the first job, i.e., the flow value of unemployment in standard search models. The model estimates the cost of working a second job to be 8 percent of average monthly earnings for men and 15 percent for women.

Third, introducing multiple jobholding into a standard on-the-job search model reduces the rate of separation from employment, increases the employment rate, and leads to a slightly higher rate of job-to-job transitions. The main mechanism driving these effects is that the option of having a second job makes workers' main job survive longer. There is a flip side of these results for the calibration of standard search models. These models underestimate the volatility of shocks to match productivity that rationalizes the volume of job separations, i.e., they would need additional volatility to hit the target if they allowed for multiple jobholding; and in order to match the job-to-job transition rate, they require a higher on-the-job search intensity relative to a world with multiple jobholding. Quantitatively, the differences are significant for the volatility of match productivity and less so for on-the-job search intensity.

Fourth and last, multiple jobholding matters for the analysis of job creation. To illustrate this point, we use the model to compute the response of job creation to changes in workers' on-the-job search intensity. We show that there are three channels that mediate this response. A higher on-the-job search intensity means a larger pool of job seekers; conditional on meeting a job seeker, a higher probability that she is employed as opposed to not having a job; and a higher joint surplus from employment. Multiple jobholding mostly affects the changing probability of meeting an employed worker. From a firm's perspective, this means a higher risk of becoming a secondary employer, which yields a substantially lower surplus relative to that of a primary employer. Thus, multiple jobholding dampens the positive feedback from higher on-the-job search intensity onto additional job creation. Lacking this mechanism, the model would overstate the capacity of on-the-job search to amplify labor market fluctuations.

Our paper contributes to several strands of literature. First and foremost, we substantially expand existing research on multiple jobholding. Much of this literature focuses on understanding the decision to hold a second job from the perspective of the classical labor supply model (Shishko and Rostker [1976], O'Connell [1979], Krishnan [1990]), 'job portfolios' models (Oaxaca and Renna [2006], Hlouskova et al. [2017]), and recently with the added equilibrium effects of education and labor productivity (Auray et al. [2021]). These analyses are conducted in static, frictionless models.[5] Paxson and Sicherman [1996] is an early exception presenting a dynamic model of multiple jobholding. However, the model is essentially illustrative and not used for quantitative inference. Two recent contributions explore the role of multiple jobholding in the context of partial-equilibrium search models. Mancino and Mullins [2022] analyze the effects of income tax incentives on labor supply, notably the decision to accept second jobs; Lo [2022] studies workers' willingness to work fewer or longer hours given exogenous offers of wage-hour bundles. As far as we are aware, our paper is the first to offer a full-fledged dynamic model of multiple jobholding cast in a general equilibrium setting.[6]

The paper is also related to a long-standing literature in macro and labor economics that studies how and to what extent individuals can adjust their working hours. Altonji and Paxson [1988, 1992] and Blundell et al. [2008], among others, show that workers often need to change job to adjust their hours. Multiple jobholding is relevant in this context because, as Paxson and Sicherman [1996] and Kahn and Lang [1991, 2001] point out, taking on a second job is an effective channel to adjust hours that may offer a valuable alternative to an employer change. Our contribution is to analyze the interplay between adjustments in hours worked, job-to-job transitions and multiple jobholding through the lens of a structural model. To illustrate the relevance of this approach, we show that multiple jobholding affects the behavior of worker flows in and out of employment, which are typically the focus of the search literature. This echoes recent research by Chang et al. [2019] who show that carefully modeling the intensive margin can materially affect our understanding of the behavior of the extensive margin.

The paper is organized as follows. Section 2 describes a few facts about multiple jobholding to help contextualize this analysis. Section 3 presents the model. Section 4 proceeds with the

[5]The literature also includes numerous studies that, without relying on a formal theoretical model, offer a wealth of empirical information useful for understanding the determinants of multiple jobholding; see Kimmel and Powell [1999], Conway and Kimmel [2001] and Panos et al. [2014].

[6]There is an analogy between the decision problem of a single agent that may hold two jobs at the same time and that of a household formed by a couple with both members searching for a job. Dey and Flinn [2008], Guler et al. [2012] and Flabbi and Mabli [2018] have developed models of joint household search. While very rich, the models in these papers have a partial equilibrium setup with exogenous wage-offer distributions. Fang and Shephard [2019] extend this class of model to a general equilibrium setup. To do so, they introduce job posting based on the canonical framework of Burdett and Mortensen [1998]. Recall that an important channel of our model is that the current primary job provides the outside option when bargaining with the outside employer. This mechanism is shut down in a job-posting model.

quantification and validation, and Section 5 uses the model to analyze the determinants of multiple jobholding. Section 6 contains the main quantitative results. Section 7 concludes.

2 Empirical facts

In this section, we describe a few basic facts about multiple jobholding and, more importantly, establish new facts showing how this affects a worker's employment conditions in her primary job. These new facts help guide the development of the model presented in Section 3.

Data and preliminary facts. We use data from the Current Population Survey (CPS) from January 1994 through March 2020. Since 1994, the CPS has been collecting information that allow to identify multiple jobholders. The survey asks respondents about the number of jobs held during the reference week, whether they usually receive a wage or salary from the primary job, and collects information on hours worked for up to two jobs. The CPS has a longitudinal component, which further allows to match respondents across consecutive surveys to detect changes in employment status, construct worker flows, and estimate transition probabilities.

We first describe a few preliminary facts about multiple jobholding based on these data:

1. On average over the sample period, 5.5 percent of employed workers hold two jobs simultaneously in a given month.[7] We call this figure the *multiple jobholding share*.

2. Workers quickly transition out of holding a second job: the expected duration of a spell of multiple jobholding is 3.2 months on average. This explains why the multiple jobholding share is not larger when taking a snapshot of the labor market in a given month. At the same time, the probability that a single jobholder in month t becomes a multiple jobholder in $t+1$ is far from trivial. It averages at 2.2 percent, which is the same order of magnitude as the job-to-job transition rate in the U.S. labor market.[8]

3. The typical multiple jobholder works full-time on her primary job and part-time on her second job.[9] Workers who combine two part-time jobs to make a full-time income only account for 17.5 percent of male multiple jobholders, and 38.3 percent of female multiple jobholders. The vast majority of multiple jobholders (over 90 percent) hold only two jobs.

4. In terms of basic observable characteristics, a major source of variation of the multiple jobholding share is education. The multiple jobholding share is more than twice higher for workers with a college degree or higher education compared to workers with less than high school education. This positive correlation may be related to the characteristics

[7] As discussed in Section 4, it is useful to consider a longer horizon to appreciate the incidence of multiple jobholding: almost 20 percent individuals work two jobs simultaneously at some point over a 1-year horizon.

[8] When looking at single jobholders, the U.S. monthly job-to-job transition rate is actually even lower than this number: it is 1.6 percent on average over the sample period (see Table 3).

[9] The primary, or main, job is the job with the greatest number of hours worked during the reference period.

of jobs held by more educated workers. They are more likely to be in professional and service occupations, which typically afford a greater flexibility in terms of work schedule.

In the rest of the analysis, we focus on workers aged 25 to 54 years old with some College or higher education. Our model features agents that are homogeneous *ex ante*, which motivates us to abstract from certain source of heterogeneity (e.g., labor force entry and exit) and to focus on a segment of the population with a higher prevalence of multiple jobholding. For consistency with the model, we also exclude individuals who report being self-employed.

Second job and same employer. Starting in 1994, the CPS measures whether individuals change employers, or change duties or activities in their main job between two consecutive months of interview. We use this information to further characterize transitions in and out of multiple jobholding. We establish the following facts (see Table B1 in Appendix B for details):

5. Almost no worker reports a change in the duties or activities of the main job upon making a transition towards (i.e., the inflow of) multiple jobholding. The same pattern holds true upon giving up the second job (i.e., the outflow).

6. Upon returning from multiple to single jobholding, only about 4 percent of individuals working full-time switch employers. For those working part-time on their main job, the corresponding figure is 8.7 percent for men and 10.4 percent for women. That is, the vast majority of workers remain with their primary employer upon giving up a second job.

These two facts point to some inertia and persistence of employment at the primary employer. In the model presented in Section 3, these features will be rationalized by the assumption that multiple jobholders do not leave their primary employer unless hit by a large adverse shock.

Main job wages and hours worked. Next, we use earnings information available for the Outgoing Rotation Groups of the CPS to consider the following type of regression:

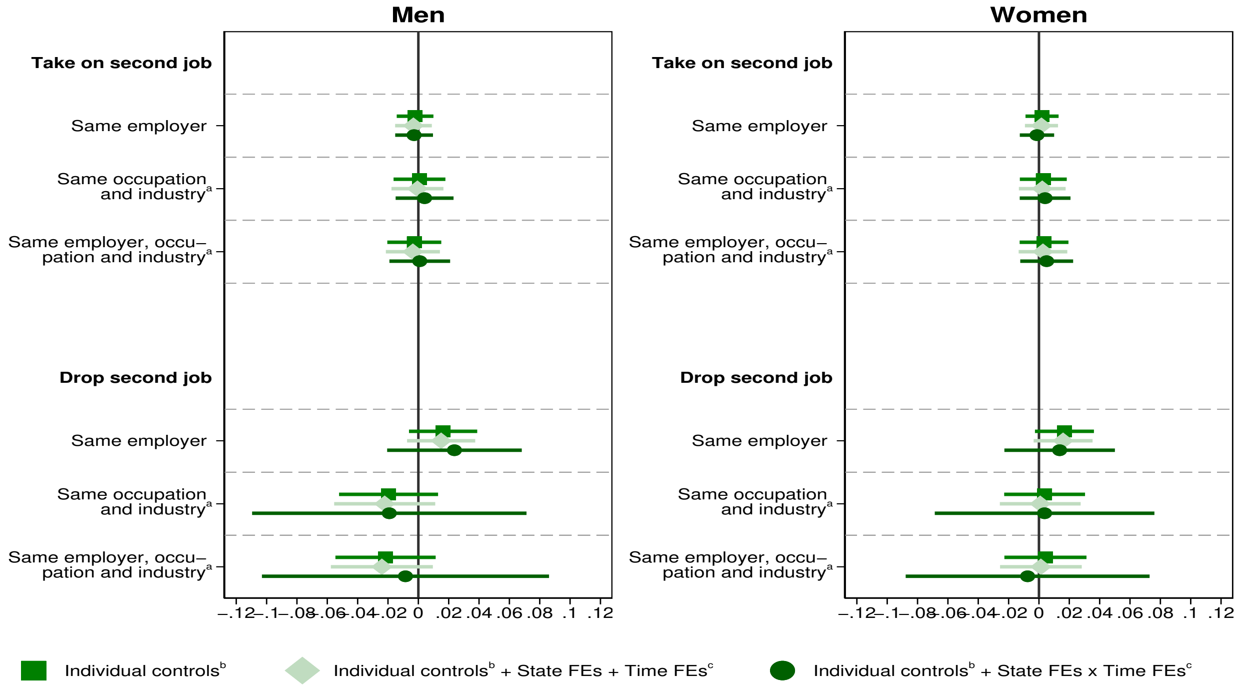
$$\triangle \log w_{i,t} = \alpha + \beta m_{i,t} + \mathbf{X}'_{\mathbf{i,t}}\gamma + \varepsilon_{i,t},$$

where $\triangle \log w_{i,t}$ is the yearly change in individual i's log wage on her primary job; $m_{i,t}$ is an indicator that takes the value of one if single jobholder i takes on a second job, and is zero otherwise; $\mathbf{X}_{\mathbf{i,t}}$ is a set of individual characteristics; and $\varepsilon_{i,t}$ is the residual. Analogously, we run the regression where $m_{i,t}$ is the indicator for dropping a second job. We consider variants of both regressions with state and time fixed effects to control for market-wide shocks. Finally, we run analogous regressions with usual hours worked as the dependent variable.[10]

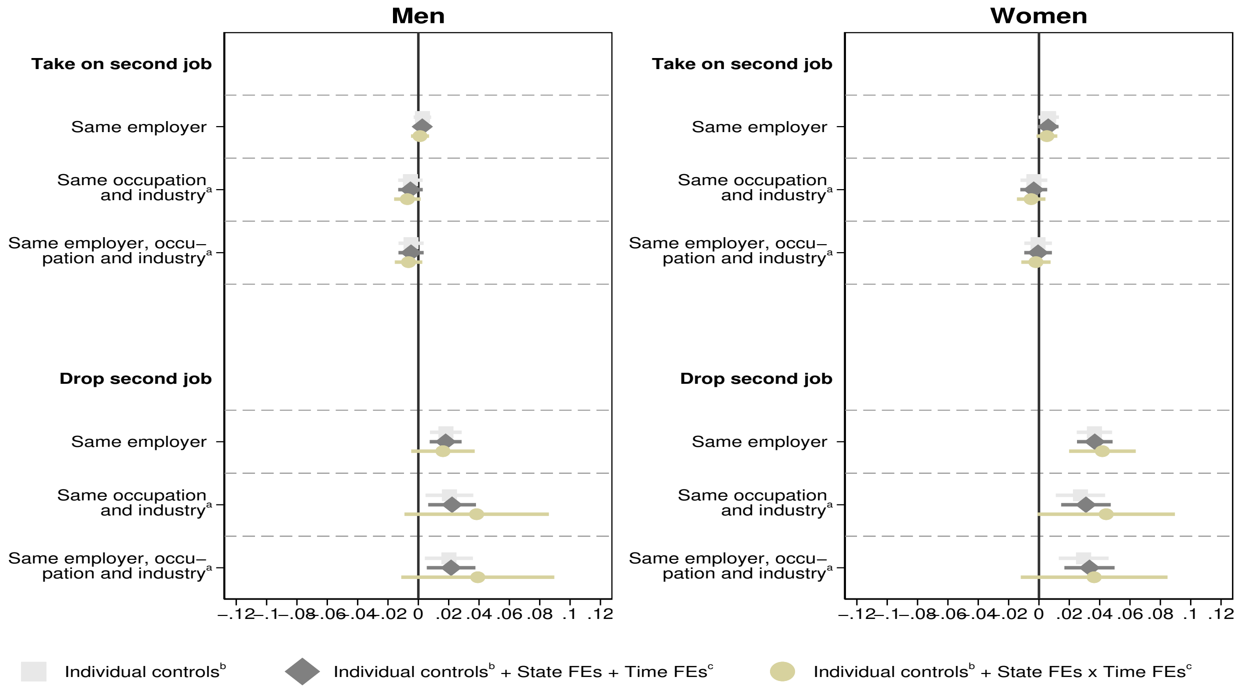
Figure 1 presents the $\widehat{\beta}$'s obtained from different regressions in which we seek to minimize other sources of changes in the dependent variable by focusing on individuals who remain with the same employer, and/or remain in the same occupation and industry of employment.[11]

[10] Actual hours worked deliver qualitatively similar results, with less precision in the estimates.

[11] The details of the empirical analysis are presented in Appendix B. We pool together data from the Outgoing Rotation Groups for the whole sample period. For the set of regressions looking at the effects of taking on a



(a) Hourly wage



(b) Usual hours worked

Figure 1: Effects of multiple jobholding on wages and hours worked of the primary job

Notes: The figure shows the estimated coefficients and 95% confidence intervals of regressing changes in log wages (Panel (a)) or changes in log hours worked (Panel (b)) on a dummy for taking on (upper set of coefficients) or dropping (lower set of coefficients) a second job. Data come from the Current Population Survey for individuals aged 25 to 54 with some College or higher education. Each regression (for respectively wages and hours worked) is run separately for men and women on three different samples: individuals who remain employed at the same employer; remain in the same occupation and industry; remain at the same employer within the same occupation and industry. ([a]) Occupations and industries are defined at the 3-digit level of the classifications provided by IPUMS-CPS (see Appendix B). ([b]) Individual controls include: a cubic polynomial of age, and education, marital and race dummies. ([c]) FEs: fixed effects.

Naturally, these correlations should not be interpreted as causal, but they provide us with a compelling observation: that taking on or giving up a second job has no statistically discernible impact on wages and hours worked of the primary job. We view this as both a motivation and supportive evidence for assuming – as we do in the model – that multiple jobholders do not use the second job to rebargain wages and hours at their primary job.

3 A search model with multiple jobholders

3.1 Economic environment

Time $t = 0, 1, \ldots$ is discrete and runs forever. The economy is populated by a unit continuum of workers and by an endogenous measure of employers, both of whom are infinitely lived and discount the future at rate $\beta^{-1} - 1$.

Workers derive utility from market and nonmarket consumptions. They seek to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_t^m + c_t^n \right). \tag{1}$$

Market consumption, c_t^m, consists of labor earnings net of a fixed cost of working ω_j, which is incurred for *each job* that the individual works. The number of jobs held is capped at two, meaning that $j \in \{1, 2\}$. Workers are endowed with one unit of time per period, and h_t denotes hours allocated to market work. Nonmarket consumption, c_t^n, consists of a home-produced good. The production of the home good depends on productivity in the home sector, z_t, which is idiosyncratic to the worker, and on the nonmarket hours of the worker, $1 - h_t$. Specifically, a strictly increasing and concave function $g\,(.)$ maps nonmarket hours onto home production, such that $c_t^n = z_t g\,(1 - h_t)$. Home productivity z_t evolves over time according to a persistent stochastic process with transition function G, i.e., $G\,(z'|z) = \Pr\{z_{t+1} < z' | z_t = z\}$. As will become clear in the sequel, the role of the home good and home production is to generate movements in a worker's hours worked beyond those triggered by shocks to the productivity of market hours.

The objective of employers is to maximize the expected present value of profit streams π_t:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \pi_t. \tag{2}$$

Each employer has at most one job that is either filled or vacant. A vacant job costs the employer κ units of output per period. A filled job produces a flow of output $y_t f\,(h_t)$, where y_t denotes idiosyncratic match productivity. $f\,(\cdot)$ is the function that maps a worker's hours worked at the current employer, h_t, onto labor services. Match productivity y_t is stochastic

second job, the sample contains almost 340,000 observations. For the set of regressions looking at the effects of giving up the second job, conditions are more restrictive as the individual must be a multiple jobholder to begin with, and we end up working with a sample of about 25,000 observations. These sample sizes explain the differences in the precision of the estimates reported in the different panels of Figure 1.

and exhibits persistence over time. Its transition function is denoted as F. Employers enter the labor market until the value of holding a vacant job equals zero.

Workers and employers come together via search. The number of contacts per unit of time depends on the number of vacancies and number of job seekers. This relationship is governed by a constant-returns-to-scale function, meaning that the job-filling probability, $q(.)$, depends only on labor market tightness θ_t, i.e., the ratio between vacancies and job seekers. Both nonemployed workers and single jobholders search for jobs. On the other hand, if a worker has two jobs, she must give up either at least one of them to start receiving job offers.[12] The probabilities that a nonemployed worker and that a single jobholder meet an employer with a vacant job are, respectively,

$$\lambda_{0,t} = \theta_t q(\theta_t) \text{ and } \lambda_{1,t} = s_e \lambda_{0,t}. \tag{3}$$

In (3), s_e measures relative on-the-job search intensity, which is a key parameter of virtually any on-the-job search model. Upon meeting, match productivity y_t is sampled from a distribution denoted as F_0. After observing the initial y_t, the worker and the employer either walk away from each other or choose to stay together.

We need certain assumptions to accommodate the option to hold two jobs and operationalize key notions related to multiple jobholding. On receiving an outside job offer, the worker can either turn down the offer, move to the new employer (job-to-job transition), or combine the new job with her current job (*multiple jobholding*). In the latter event:

(A1) The worker can give up the second job at any time, but until she does so, she cannot quit her older employer – called *primary employer* –, unless their match gets dissolved.

(A2) Her wages and hours at the primary employer are determined by the same bargain as that of a single jobholder who would have the same match and idiosyncratic home productivity.

(A3) The worker can use the primary job as her outside option to bargain on wages and hours with the new employer – henceforth called the *secondary employer*.

These assumptions are restrictive along some dimensions, but as we will point out in the next sections, they provide a substantial benefit in terms of tractability. It is also clear that they have bearing on the predictions of the quantitative model concerning turnover in and out of second jobs. We will show in Section 4 that these predictions come remarkably close to the data, even though the calibration targets none of the inflows and outflows of second jobs. This, together with facts presented in the previous section, is evidence that Assumptions (A1)–(A3) are empirically relevant to our understanding of multiple jobholding.

Let us make a few additional remarks to conclude this section. First, we have mentioned worker-firm bargaining in (A2)–(A3). The bargaining protocol that will be set up in the next

[12] We could allow multiple jobholders to search on the job, but this would only complicate the model without adding value for our understanding of multiple jobholding. As described in Section 2, the vast majority of multiple jobholders hold only two jobs, and there are only few job-to-job transitions among multiple jobholders.

sections is such that participation constraints are satisfied on both employers' sides. Second, as in Pissarides [1994] and Fujita and Ramey [2012], we use the timing convention that if the worker opts for a job-to-job transition, then she immediately loses contact with the former employer. As a result, the worker cannot use her previous employment status to bargain for a higher wage at the new employer. Vice versa, if the worker discards the option of moving to the outside employer, then this contact is lost immediately and her bargaining position remains unchanged at the current employer. Third, we exclude the possibility for the secondary employer of inducing a quit by paying a wage bonus upon meeting the worker.[13]

3.2 Asset values and joint match surpluses

From this point on, we drop the time subscript t and use a recursive formulation of agents' decision problems. We denote by $N(z)$, $E(y_1, z)$ and $E(y_1, y_2, z)$ the asset values of, respectively, nonemployed workers, single jobholders, and multiple jobholders. For firms, we use $J(y_1, z)$ to denote the asset value of an employer matched to a single jobholder. For those matched to a multiple jobholder, we denote by $J_1(y_1, y_2, z)$ and $J_2(y_1, y_2, z)$ the asset values of the primary and secondary employers, respectively. The asset value of holding a vacancy is always zero.

There are two joint match surpluses to be defined. The first one is the joint match surplus of employment with a single jobholder, $S(y_1, z)$, which is also the surplus of the primary job held by a multiple jobholder following Assumption (A2). The surplus is given by

$$S(y_1, z) = J(y_1, z) + E(y_1, z) - N(z). \tag{4}$$

Following Assumption (A3), the surplus of the second job of a multiple jobholder, denoted as $S(y_1, y_2, z)$, is defined by

$$S(y_1, y_2, z) = J_2(y_1, y_2, z) + E(y_1, y_2, z) - E(y_1, z). \tag{5}$$

That is, the surplus of multiple jobholding consists of the surplus of the secondary employer and the worker's asset value of holding two jobs net of the value of working only on her primary job. Note that $J_1(y_1, y_2, z)$ does not appear in the system of equations (4) and (5). $J_1(y_1, y_2, z)$ will show up only in the continuation value of $J(y_1, z)$, and thus in the joint surplus $S(y_1, z)$.

3.3 Bargaining

As already mentioned, workers and employers bargain on wages and hours period by period to split the surplus. Formally, the asset values introduced above are defined as

$$E(y_1, z) \equiv \tilde{E}(w(y_1, z), h(y_1, z); y_1, z) \quad \text{and} \quad J(y_1, z) \equiv \tilde{J}(w(y_1, z), h(y_1, z); y_1, z), \tag{6}$$

[13]We similarly rule out this type of side payments in any future periods as the secondary employer must recognize that the worker is committed to her primary employer.

where $w(y_1, z)$ and $h(y_1, z)$ are the wages and hours worked resulting from Nash bargaining (detailed below); and, similarly, the asset values $E(y_1, y_2, z)$ and $J_2(y_1, y_2, z)$ are defined as

$$E(y_1, y_2, z) \equiv \tilde{E}(w(y_1, y_2, z), h(y_1, y_2, z); y_1, y_2, z)$$
$$\text{and} \;\; J_2(y_1, y_2, z) \equiv \tilde{J}_2(w(y_1, y_2, z), h(y_1, y_2, z); y_1, y_2, z), \qquad (7)$$

with $w(y_1, y_2, z)$ and $h(y_1, y_2, z)$ resulting from Nash bargaining. The derivation of $\tilde{E}(w, h; y_1, z)$, $\tilde{E}(w, h; y_1, y_2, z)$, $\tilde{J}(w, h; y_1, z)$, $\tilde{J}_2(w, h; y_1, y_2, z)$ is provided in Appendix A.3.

Let $\phi \in (0, 1)$ denote workers' Nash bargaining share. The solution of

$$\max_{w,h} \left\{ \left(\tilde{E}(w, h; y_1, z) - N(z) \right)^{\phi} \tilde{J}(w, h; y_1, z)^{1-\phi} \right\} \qquad (8)$$

yields wages and hours for single jobholders, as well as for the primary job of multiple jobholders under Assumption (A2). Note that since $J_1(y_1, y_2, z)$ is included in the continuation value of $J(y_1, z)$, the wage schedule that comes out of (8) accounts for the fact that a worker may eventually become a multiple jobholder and that this would not change her bargaining position vis-à-vis the primary employer.

Next, following (A3), wages and hours worked for the second job are pinned down by:

$$\max_{w,h} \left\{ \left(\tilde{E}(w, h; y_1, y_2, z) - E(y_1, z) \right)^{\phi} \tilde{J}_2(w, h; y_1, y_2, z)^{1-\phi} \right\}, \qquad (9)$$

subject to $J(y_1, z) \geq 0$, i.e. the worker cannot retain the primary job if this job is not viable. For reasons discussed in Section 3.4, it is also the case in equilibrium that $J_1(y_1, y_2, z) \geq J(y_1, z)$, so that this employer is not worse off when her worker chooses to hold a second job. The solution of (9) ensures participation of the secondary employer, as explained in Proposition 1:

Proposition 1. *Wages in primary and second jobs split the surplus in proportion to the agents' bargaining weights:*

$$E(y_1, z) - N(z) = \phi S(y_1, z) \;\; \text{and} \;\; J(y_1, z) = (1 - \phi) S(y_1, z), \qquad (10)$$

$$E(y_1, y_2, z) - E(y_1, z) = \phi S(y_1, y_2, z) \;\; \text{and} \;\; J_2(y_1, y_2, z) = (1 - \phi) S(y_1, y_2, z). \qquad (11)$$

Since $E(y_1, y_2, z) - E(y_1, z) \geq 0 \Rightarrow J_2(y_1, y_2, z) \geq 0$*, the participation constraint of the secondary employer is satisfied.*

If the functions $f(.)$ *and* $g(.)$ *are differentiable, then at an interior solution, hours worked in primary and second jobs satisfy, respectively,*

$$y_1 f'(h(y_1, z)) = z g'(1 - h(y_1, z)), \qquad (12)$$

$$y_2 f'(h(y_1, y_2, z)) = z g'(1 - h(y_1, z) - h(y_1, y_2, z)). \qquad (13)$$

Hence, hours worked equalize the marginal product in the market and the home sector.

Proof. See Appendix A.1. □

Note that the Proposition discusses $J_2(y_1, y_2, z) \geq 0$, but it does not compare it to the surplus $J(y_2, z)$ that the secondary employer would get if the worker would not have a primary job. It is intuitive that $J(y_2, z)$ is higher than $J_2(y_1, y_2, z)$ for most y_2 and z, since the worker has a better outside option compared to that of a single jobholder.[14] Besides this, a second job generates little output, given the immediate corollary of Equations (12)-(13):

Corollary. *Ceteris paribus, a multiple jobholder works fewer hours on her second job compared to a single jobholder employed in that job, i.e.,*

$$h(y_1, y_2, z) \leq h(y_2, z) \tag{14}$$

for two workers with the same idiosyncratic home component z and match productivity y_2, regardless of match productivity y_1 of the multiple jobholder in her primary job.

The corollary follows directly from the observation that hours worked in the primary job reduce the marginal productivity in the home sector – equivalently, that hours worked in the second job come in at a higher marginal disutility of work.

A corollary to this corollary is that a multiple jobholder is likely to work more hours on her primary job than in her second job. To see why, consider – unlike in the corollary above – hours of the *same* individual, so that z is fixed. If $y_2 \approx y_1$, then $h(y_1, z) \approx h(y_2, z) \geq h(y_1, y_2, z)$ following Equation (14).[15] $y_2 \approx y_1$ is a reasonable approximation for multiple jobholders because on the one hand if y_2 were much higher than y_1 then the worker would have quit to the outside employer upon meeting; and on the other hand, if y_2 is much lower than y_1, then the second job generates too little surplus to cover the cost of working a second job.

3.4 Bellman equations[16]

We need to first define some policy functions to write the Bellman equations. Proposition 2 below enables us to focus on three functions that correspond to the following binary decisions: (i) an employer's decision to keep a job alive, $p(y_1, z) = \mathbb{1}\{J(y_1, z) \geq 0\}$; (ii) a worker's decision to take on a second job, $d(y_1, y_2, z) = \mathbb{1}\{E(y_1, y_2, z) \geq E(y_1, z)\}$; (iii) a worker's decision to leave the current job upon meeting an incumbent employer, $\ell(y_1, y_2, z) = \mathbb{1}\{\max\{E(y_2, z), N(z)\} \geq \max\{E(y_1, z), E(y_1, z) + p(y_1, z)(E(y_1, y_2, z) - E(y_1, z)), N(z)\}\}$. Observe that inside the 'max' operator $p(y_1, z)$ multiplies $E(y_1, y_2, z) - E(y_1, z)$ as per Assumption (A1): the option of having a second job is contingent on the first job being viable.

[14] Thus, in most instances the secondary employer would be willing to pay a fee to the worker to induce her to quit her primary job, but Assumption (A1) rules out this type of side payments.

[15] This implication of the model is fully consistent with the empirical definition of the primary job, which is defined as the job that has the greatest number of hours worked; see Section 2.

[16] In this section, we focus on the Bellman equations for $S(y_1, z)$, $S(y_1, y_2, z)$, and $J_1(y_1 y_2, z)$, because these asset values are sufficient to describe the equilibrium of the model. $S(y_1, z)$ and $S(y_1, y_2, z)$ are derived from the Bellman equations that define $N(z)$, $E(y_1, z)$, $E(y_1, y_2, z)$, $J(y_1, z)$, $J_1(y_1, y_2, z)$, $J_2(y_1, y_2, z)$ through calculations presented in Appendix A.3.

Proposition 2. *The policy functions* $p(y_1, z)$, $d(y_1, y_2, z)$, $\ell(y_1, y_2, z)$ *can be expressed jointly as functions of the joint match surpluses* $S(y_1, z)$ *and* $S(y_1, y_2, z)$. *Specifically,*

$$p(y_1, z) = \mathbb{1}\{S(y_1, z) \geq 0\} \tag{15}$$

$$d(y_1, y_2, z) = \mathbb{1}\{S(y_1, y_2, z) \geq 0\} \tag{16}$$

$$\ell(y_1, y_2, z) = \mathbb{1}\{p(y_2, z) S(y_2, z) \geq p(y_1, z)(S(y_1, z) + d(y_1, y_2, z) S(y_1, y_2, z))\} \tag{17}$$

Proof. See Appendix A.2. □

With these policy functions at hand, we can describe the Bellman equations of the joint surpluses, $S(y_1, z)$, $S(y_1, y_2, z)$, and the asset value of a primary employer, $J_1(y_1, y_2, z)$. To simplify notations, we also include $N(z)$, the value of being nonemployed, in the system of equations below. $N(z)$ solves[17]

$$N(z) = \beta \int \left(N(z') + \lambda_0 \phi \int p(y_1', z') S(y_1', z') \, dF_0(y_1')\right) dG(z'|z). \tag{18}$$

The continuation value of nonemployment includes the surplus of becoming a single jobholder multiplied by the worker's bargaining power.

The joint surplus of employment with a single jobholder is

$$\begin{aligned} S(y_1, z) = {} & y_1 f(h(y_1, z)) + zg(1 - h(y_1, z)) - (N(z) + \omega_1) + \beta \Bigg(S_e^+(y_1, z) + S_j^+(y_1, z) \\ & + \int \left(\int p(y_1', z') \left(1 - \lambda_1 \int \ell(y_1', y_2', z') \, dF_0(y_2')\right) S(y_1', z')\right) dF(y_1'|y_1) \Bigg) dG(z'|z) \Bigg) \end{aligned} \tag{19}$$

where

$$\begin{aligned} S_e^+(y_1, z) = {} & \int \Bigg(N(z') + \phi \lambda_1 \int \int (\ell(y_1', y_2', z') p(y_2', z') S(y_2', z') + (1 - \ell(y_1', y_2', z')) \\ & \times p(y_1', z') d(y_1', y_2', z') S(y_1', y_2', z')) \, dF_0(y_2') \, dF(y_1'|y_1) \Bigg) dG(z'|z) \end{aligned} \tag{20}$$

and

$$\begin{aligned} S_j^+(y_1, z) = {} & \lambda_1 \int \int \int ((1 - \ell(y_1', y_2', z')) p(y_1', z') d(y_1', y_2', z') (J_1(y_1', y_2', z') \\ & - (1 - \phi) S(y_1', z'))) \, dF_0(y_2') \, dF(y_1'|y_1) \, dG(z'|z). \end{aligned} \tag{21}$$

There are three components in the continuation value of a match with a single jobholder. The first one is the worker's component $S_e^+(y_1, z)$ defined in Equation (20), which captures the option that a single job allows the worker eventually to switch employers or to take on a second job. Second, the employer's component $S_j^+(y_1, z)$ shown in Equation (21) captures, for the incumbent firm, the effect of becoming the primary employer if the worker takes on a second job, the net surplus of which is $J_1(y_1, y_2, z) - (1 - \phi) S(y_1, z)$. Third, if the worker neither

[17] Any flow value of nonemployment is subsumed in the ω_j's, the flow cost of working; see Table 5 for details.

leaves nor takes a second job, then in Equation (19) the worker-firm pair receives the surplus $S(y_1, z)$ in the subsequent period if the job is kept alive.

Next, consider the match surplus between a secondary employer and a multiple jobholder, $S(y_1, y_2, z)$. Its asset value is given by

$$
\begin{aligned}
S(y_1, y_2, z) &= y_2 f(h(y_1, y_2, z)) + zg(1 - h(y_1, z) - h(y_1, y_2, z)) - \omega_2 \\
&\quad - (\phi S(y_1, z) + N(z) + \omega_1 - w(y_1, z)) + \beta \Bigg(S_e^{+}(y_1, y_2, z) + \int \Bigg(\int \int p(y_1', z') \\
&\quad \times d(y_1', y_2', z') S(y_1', y_2', z') dF(y_1'|y_1) dF(y_2'|y_2) \\
&\quad + \left(\int (1 - p(y_1', z')) dF(y_1'|y_1) \right) \left(\int p(y_2', z') S(y_2', z') dF(y_2'|y_2) \right) \Bigg) dG(z'|z) \Bigg) \qquad (22)
\end{aligned}
$$

where

$$
S_e^{+}(y_1, y_2, z) = \int \left(N(z') + \phi \int p(y_1', z') S(y_1', z') dF(y_1'|y_1) \right) dG(z'|z). \qquad (23)
$$

In the continuation value of $S(y_1, y_2, z)$, $S_e^{+}(y_1, y_2, z)$ defined in Equation (23) captures the option value of the worker returning to single jobholding at her primary employer, which happens when the worker terminates the multiple jobholding spell. The remaining part in Equation (22) shows that the employment relationship may continue as a spell of multiple jobholding or evolve into single employment at the secondary employer (who would then become the sole employer of the worker).

Last, the asset value of being the primary employer of a multiple jobholder solves

$$
\begin{aligned}
J_1(y_1, y_2, z) &= y_1 f(h(y_1, z)) - w(y_1, z) + \beta \int \int p(y_1', z') \Bigg((1 - \phi) S(y_1', z') + \int (d(y_1', y_2', z') \\
&\quad \times (J_1(y_1', y_2', z') - (1 - \phi) S(y_1', z')) dF(y_2'|y_2) \Bigg) dF(y_1'|y_1) dG(z'|z). \qquad (24)
\end{aligned}
$$

That is, in the following period the employer becomes the only employer of the worker if the worker gives up her second job. Otherwise, she continues as her primary employer and receives the net surplus value $J_1(y_1, y_2, z) - (1 - \phi) S(y_1, z)$.

At this point, it is useful to compare $J_1(y_1, y_2, z)$ and $(1 - \phi) S(y_1, z) = J(y_1, z)$ – its full expression is in Appendix A.3. First, the profit flows $y_1 f(h(y_1, z)) - w(y_1, z)$ are the same due to Assumption (A2). Second, in the continuation value of $J_1(y_1, y_2, z)$, $(1 - \phi) S(y_1, z)$ is only subjected to $p(y_1, z) = 1$ – the job remains viable –, whereas in the continuation value of $J(y_1, z)$ it is also multiplied by $1 - \lambda_1 + \lambda_1 (1 - \ell(y_1, y_2, z)) \leq 1$ – the worker did not receive an outside job offer, or she received one but rejected it. Third, $J_1(y_1, y_2, z) - (1 - \phi) S(y_1, z)$ in Equation (24) is multiplied by $p(y_1, z) d(y_1, y_2, z)$ whereas in the continuation value of $J(y_1, z)$ it is multiplied by $\lambda_1 (1 - \ell(y_1, y_2, z)) p(y_1, z) d(y_1, y_2, z) \leq 1$. These expressions are the direct consequence of Assumption (A1). Given that the profit flows are the same and that job turnover is higher for single vs. multiple jobholders, we would expect $J_1(y_1, y_2, z) \geq J(y_1, z)$ to hold. The reason we cannot guarantee this result *ex ante* is that y_2' is drawn from $F(y_2'|y_2)$ in the continuation value of $J_1(y_1, y_2, z)$, whereas it is drawn from $F_0(.)$ in the continuation value of $J(y_1, z)$. If $F_0(.)$ first-order stochastically dominates $F(.|y_2)$, then this could reverse the comparison between $J_1(y_1, y_2, z)$ and $J(y_1, z)$. This never happens in the equilibrium of the calibrated model, however. In the calibrated equilibrium, $F_0(.)$ dominates $F(.|y_2)$ only for values of match productivity y_2 that are lower than average match productivity, but at these values of y_2 we have $d(y_1, y_2, z) = 0$ since the second job does not generate enough surplus to cover the flow cost of working an additional job, ω_2.

To compute the joint surplus value of multiple jobholding (Equation (22)), we need to determine the wage of a single jobholder, $w(y_1, z)$. From the asset value of employing a single jobholder, it follows that

$$w(y_1, z) = y_1 f(h(y_1, z)) - (1-\phi) S(y_1, z) + \beta \Bigg(S_j^+(y_1, z) + (1-\phi) \times \int \left(\int p(y_1', z') \left(1 - \lambda_1 \int \ell(y_1', y_2', z') \, dF_0(y_2') \right) S(y_1', z') \right) dF(y_1'|y_1) \Bigg) dG(z'|z) \Bigg) \quad (25)$$

for all y_1 and z. As anticipated, $w(y_1, z)$ includes the expected value of the worker becoming a multiple jobholder without changes to her bargaining position vis-à-vis the primary employer, through the term $S_j^+(y_1, z)$ (see Equation (21)). We can also recover $w(y_1, y_2, z)$, the wage of a multiple jobholder, by using the asset value of secondary employers. $w(y_1, y_2, z)$ is given by

$$\begin{aligned} w(y_1, y_2, z) = y_2 f(h(y_1, y_2, z)) - (1-\phi) S(y_1, y_2, z) + \beta(1-\phi) \int \Bigg(\int \int p(y_1', z') \\ \times d(y_1', y_2', z') S(y_1', y_2', z') \, dF(y_1'|y_1) \, dF(y_2'|y_2) \\ + \left(\int (1 - p(y_1', z')) \, dF(y_1'|y_1) \right) \left(\int p(y_2', z') S(y_2', z') \, dF(y_2'|y_2) \right) \Bigg) dG(z'|z) \end{aligned} \quad (26)$$

for all y_1, y_2 and z.

3.5 Free entry condition

To write the free entry condition, we let $\varphi_0(z)$ and $\varphi_1(y_1, z)$ denote the population measure of nonemployed workers and single jobholders, respectively. Below we will denote by $\varphi_2(y_1, y_2, z)$ the population measure of multiple jobholders. The free entry condition yields

$$\begin{aligned} \frac{\kappa}{q(\theta)} = \beta(1-\phi) \Bigg(\int \int p(y_1', z') S(y_1', z') \, dF_0(y_1') \, dG(z'|z) \frac{\varphi_0(z)}{\bar{\varphi}_0 + s_e \bar{\varphi}_1} dz \\ + \int \int \int S_j^+(y_1', y_2', z') \, dF_0(y_2') \, dF(y_1'|y_1) \, dG(z'|z) \frac{s_e \varphi_1(y_1, z)}{\bar{\varphi}_0 + s_e \bar{\varphi}_1} dy_1 dz \Bigg) \end{aligned} \quad (27)$$

where

$$\begin{aligned} S_j^+(y_1, y_2, z) = \ell(y_1, y_2, z) \, p(y_2, z) \, S(y_2, z) \\ + (1 - \ell(y_1, y_2, z)) \, p(y_1, z) \, d(y_1, y_2, z) \, S(y_1, y_2, z). \end{aligned} \quad (28)$$

$(1-\phi)\, S_j^+ (y_1, y_2, z)$ denotes the asset value of an employer with a vacant position who meets an employed worker. She takes as given the decision of the worker to leave the previous employer or to combine the two jobs together. In Equation (27), $\bar{\varphi}_0$ is the cumulated measure of nonemployed workers, i.e., $\bar{\varphi}_0 = \int \varphi_0 (z)\, dz$. Likewise, $\bar{\varphi}_1$ is the cumulated measure of single jobholders. $\bar{\varphi}_0 + s_e \bar{\varphi}_1$ gives the number of job seekers, which is used to obtain the conditional distribution on the right-hand side of Equation (27) (and to compute tightness $\theta = {}^{v}/_{(\bar{\varphi}_0 + s_e \bar{\varphi}_1)}$, with v denoting the measure of vacancies).

3.6 Equilibrium

The steady-state equilibrium of this economy is defined as follows:

Definition. A steady-state equilibrium is a list of asset values $N(z)$, $E(y_1, z)$, $E(y_1, y_2, z)$, $J(y_1, z)$, $J_1(y_1, y_2, z)$, $J_2(y_1, y_2, z)$; a list of wage schedules $w(y_1, z)$, $w(y_1, y_2, z)$ and schedules of hours worked $h(y_1, z)$, $h(y_1, y_2, z)$; a list of policy functions for match formation and continuation, $p(y_1, z)$, multiple jobholding $d(y_1, y_2, z)$ and quit decisions $\ell(y_1, y_2, z)$; a population distribution $\varphi_0(z)$, $\varphi_1(y_1, z)$, $\varphi_2(y_1, y_2, z)$; and a value of tightness θ such that:

1. Given wages $w(y_1, z)$, $w(y_1, y_2, z)$ and schedules of hours $h(y_1, z)$, $h(y_1, y_2, z)$, the policy functions $p(y_1, z)$, $d(y_1, y_2, z)$, $\ell(y_1, y_2, z)$, and tightness θ, the asset values $N(z)$, $E(y_1, z)$, $E(y_1, y_2, z)$, $J(y_1, z)$, $J_1(y_1, y_2, z)$, $J_2(y_1, y_2, z)$ solve the Bellman equations that sum up to (19), (22) and (24) through the surplus sharing Equations (10) and (11).

2. Given the asset values $N(z)$, $E(y_1, z)$, $E(y_1, y_2, z)$, $J(y_1, z)$, $J_1(y_1, y_2, z)$, $J_2(y_1, y_2, z)$, and tightness θ, the wage schedules $w(y_1, z)$, $w(y_1, y_2, z)$ yield the surplus sharing equations (10) and (11), and the schedules of hours worked $h(y_1, z)$, $h(y_1, y_2, z)$ solve Equations (12) and (13), respectively.

3. Given the asset values $N(z)$, $E(y_1, z)$, $E(y_1, y_2, z)$, $J(y_1, z)$, $J_2(y_1, y_2, z)$ combined into joint surpluses via Equations (4) and (5), the policy functions $p(y_1, z)$, $d(y_1, y_2, z)$, $\ell(y_1, y_2, z)$ are given by Equations (15), (16) and (17), respectively.

4. Given the policy functions $p(y_1, z)$, $d(y_1, y_2, z)$, $\ell(y_1, y_2, z)$, and tightness θ, the population distribution $\varphi_0(z)$, $\varphi_1(y_1, z)$, $\varphi_2(y_1, y_2, z)$ is time invariant with respect to the set of stock-flow equations of the economy.

5. Given the asset values $J(y_1, z)$ and $J_2(y_1, y_2, z)$ combined into joint match surpluses through Equations (10) and (11), and population distribution $\varphi_0(z)$, $\varphi_1(y_1, z)$, labor-market tightness θ solves Equation (27).

The stock-flow equations across the different states of nature (condition 4 in the above definition) can be deduced from the model's description. Given a population distribution $\varphi_0(z)$, $\varphi_1(y_1, z)$, $\varphi_2(y_1, y_2, z)$ and a value of market tightness θ, Proposition 2 enables us to compute the equilibrium by solving Equations (19), (22), (24) (for instance using value-function iterations) while recovering the wage schedule $w(y_1, z)$ via Equation (25).

4 Quantification and model validation

Next, we turn the theoretical framework presented in the previous section into a fully quantitative tool. This is a key step of our analysis, given that the motivation and the questions that it addresses are quantitative in nature.

4.1 Specification

We begin with the specification of stochastic processes. As is standard, we assume that match productivity y evolves according to a first-order autoregressive process. We denote by μ_y the unconditional mean of the process, $\rho_y \in (0,1)$ the persistence, and σ_ε^2 the variance of the innovation term.[18] For simplicity we use $F_0(\cdot) = F(\cdot|\mu_y)$ for the distribution from which y is drawn upon meeting. Next, we set up a stochastic process for home productivity, z, defined in the following way: with probability ρ_z the value of z is unchanged, while with probability $1-\rho_z$ a new z' is drawn from a Normal distribution $\mathcal{N}(\mu_z, \sigma_z^2)$.[19] We discretize and truncate this distribution to the interval $[\mu_z - 2\sigma_z, \mu_z + 2\sigma_z]$.

Next, we need a production function for the home good, $g(.)$, and a function mapping a worker's hours worked to her market's labor services, $f(.)$. For the former, we use a standard specification that we can relate to studies from the labor supply literature:

$$g(1-h) = \frac{(1-h)^{1-\frac{1}{\gamma}} - 1}{1-\frac{1}{\gamma}}. \tag{29}$$

For $f(.)$, we propose a less standard, but intuitive and parsimonious specification. We use the following piecewise linear function:

$$f(h) = \begin{cases} (1-\psi)h & \text{if } h < \bar{h} \\ (1-\psi)h + \psi & \text{if } h \geq \bar{h} \end{cases}, \tag{30}$$

where $\psi \geq 0$ and $\bar{h} > 0$ are exogenous parameters. This simple construct generates a meaningful distinction between full-time and part-time employment (later defined as hours worked below the $\bar{h}$ threshold) which will prove very useful for making connections to the data.[20] Indeed, in (30), the marginal increment in labor services coming from additional hours worked is constant except for some neighborhood to the left of $\bar{h}$. Since a small increase in hours worked in that neighborhood – the cost of which is a small reduction in home production – generates a

[18] Given a pre-specified number of grid points to discretize the support of match productivity, we approximate the transition function $F(y'|y)$ using Rouwenhorst [1995]'s method.

[19] In Lalé [2018], we use a similar shock process for workers' preferences for leisure to generate worker flows in and out of the labor force. From a computational standpoint, the advantage of this stochastic process is that, conditional on changing z, the new z' is drawn independently from the previous z.

[20] The non-convex mapping from hours worked to labor services in (30) shares similarities with that proposed by Prescott et al. [2009], Rogerson and Wallenius [2009] and Chang et al. [2019] to distinguish between the extensive and intensive margins of labor adjustments. It also parallels with the mapping from hours to earnings studied in Bick et al. [2022].

discrete increase by ψ units in labor services, this should 'bunch' working hours towards $\bar{h}$. The Proposition below confirms this intuition.

Proposition 3. *The hours schedule in a primary job is given by:*

$$h(y_1, z) = \begin{cases} \bar{h} & \text{if } y_{\bar{h}}(z) \leq y_1 < \widetilde{y}(z) \\ 1 - \left(\frac{z}{(1-\psi)y_1}\right)^{\gamma} & \text{otherwise,} \end{cases} \tag{31}$$

for all (strictly positive) y_1 and z. $y_{\bar{h}}(z)$ and $\widetilde{y}(z)$ are functions presented in the Appendix.

The hours schedule for the second job of multiple jobholders is:

$$h(y_1, y_2, z) = \begin{cases} \bar{h} & \text{if } y_{\bar{h}}(y_1, z) \leq y_2 < \widetilde{y}(y_1, z) \\ 1 - h(y_1, z) - \left(\frac{z}{(1-\psi)y_2}\right)^{\gamma} & \text{otherwise,} \end{cases} \tag{32}$$

for all (strictly positive) y_1, y_2, and z. $y_{\bar{h}}(y_1, z)$ and $\widetilde{y}(y_1, z)$ are functions presented in the Appendix.

Proof. See Appendix A.4. □

Consider the hours schedule of a single jobholder, or similarly that of the primary job of a multiple jobholder, described in (31). Hours worked increase in a concave fashion with match productivity y_1. At $y_1 = y_{\bar{h}}(z)$, agents are indifferent between setting hours worked at $1 - \left(\frac{z}{(1-\psi)y_{\bar{h}}(z)}\right)^{\gamma} < \bar{h}$ vs. setting hours at $\bar{h}$, and the hours schedule jumps up to $\bar{h}$. For any y_1 between $y_{\bar{h}}(z)$ and $\widetilde{y}(z)$, the hours schedule is flat as it is optimal to keep hours at $\bar{h}$. For any $y_1 \geq \widetilde{y}(z)$, hours increase again with y_1 through $h(y_1, z) = 1 - \left(\frac{z}{(1-\psi)y_1}\right)^{\gamma}$. Notice that hours decrease with home productivity, z, and that hours worked in the primary job $h(y_1, z)$ reduce hours worked in the second job, $h(y_1, y_2, z)$.

To complete the model's specification, we need a matching function to map market tightness into the job-filling probability. We use a standard Cobb-Douglas matching function, such that the number of contacts per unit of time is $\chi v^{1-\alpha}(\bar{\varphi}_0 + s_e\bar{\varphi}_1)^{\alpha}$, where v denotes vacancies and $\bar{\varphi}_0 + s_e\bar{\varphi}_1$ is the number of job seekers. The job-filling probability is then $q(\theta) = \chi\theta^{-\alpha}$.

4.2 Calibration

For reasons discussed further below, we add to the model a share of exogenous job separations to discipline certain parameters. We assume that all jobs are destroyed with a per-period probability δ; conditional on not being hit by the δ shock, the events unfold as described in Section 3. Given these specification choices, the model has eighteen parameters:

$$\beta,\ \bar{h},\ \mu_y,\ \rho_y,\ \alpha,\ \phi,\ \chi,\ \gamma,\ \mu_z,\ \rho_z,\ \sigma_z,\ \psi,\ \kappa,\ \sigma_{\varepsilon},\ \delta,\ s_e,\ \omega_1,\ \omega_2.$$

We use external information to select parameter values for the first seven of these parameters. We calibrate the other parameters to match several data moments, most of which are based on CPS data analyzed in Appendix B. Given these data, the model period is set to one month.

Externally calibrated parameters. We use a discount factor β of 0.9951 to accord with an annualized real interest rate of 6 percent. To choose $\bar{h}$, we note there are about 100 hours of substitutable time per week and the standard full-time work schedule amounts to 40 hours of work per month. So, we set $\bar{h} = 0.40$ given that the time endowment of workers has been normalized to 1 (Equation (29)). We can have one more normalization, namely the unconditional mean of match productivity, μ_y, which we set to 1. Next, for the persistence of match productivity ρ_y, we follow much of the literature and interpret the empirical observation that wage shocks are highly persistent as evidence that match productivity is close to unit root. Hence, we use $\rho_y = 0.975$. At an annual frequency, this implies a persistence of $0.975^{12} = 0.738$, which falls well within the range of estimates of wage processes.[21] We set the elasticity of the vacancy-filling probability with respect to labor market tightness α and the bargaining power of workers ϕ equal to 0.5. Finally, the matching efficiency parameter χ is set to 0.50. Below we calibrate the vacancy-posting cost and use the free entry condition to pin down labor market tightness. Table 1 provides the list of externally calibrated parameters.

Internally calibrated parameters. The remaining parameters, namely γ, μ_z, ρ_z, σ_z, ψ, κ, σ_ε, δ, s_e, ω_1, ω_2, are calibrated to match eleven data moments. Although these are jointly determined, each parameter is more directly associated to a specific data moment, as we discuss momentarily. Given the strong relation between multiple jobholding and full-time/part-time employment on the one hand, and the relation between part-time employment and gender on the other, we calibrate the parameters separately for men (M) and women (W). We focus on individuals aged 25 to 54 years old with some College or higher education to reduce the amount of heterogeneity in the underlying data. For each gender group, we target:

1. The **Frisch elasticity of labor supply**. Given (29), the Frisch elasticity is

$$\mathcal{F} = \gamma \frac{1-h}{h}. \tag{33}$$

   We consider different values for $\mathcal{F}$, namely we use $\mathcal{F} = 0.45$ as our benchmark but we also study $\mathcal{F} = 0.30$ and $\mathcal{F} = 0.60$ to cover the range of plausible values (see Table 3C in Meghir and Phillips [2010] or Table 1 in Keane [2011]). Besides the uncertainty as to the precise value of the Frisch elasticity, we find that exploring the effects of different values of $\mathcal{F}$ is interesting in its own rights.

2–5. The **part-time employment share**, the **transition rate from full- to part-time** work hours, **average hours per worker**, and the share of workers **bunching at full-time hours**. We target four data moments on hours worked, three of which describe the cross-sectional distribution of working hours and one describing worker transitions within this distribution.[22] Since a worker's hours schedule is negatively related to her

[21] See for instance in Table 1 in Chang and Kim [2006]: the authors report an annual persistence of wage shocks (estimated from PSID data) of 0.781 for men and 0.724 for women.

[22] We need moments besides the mean and variance to characterize the empirical distribution of working hours, given that this distribution exhibits multiple modes, clustering, etc. (Borowczyk-Martins and Lalé [2019]).

current z (from Proposition 3), the part-time employment share (defined in the model as the share of employment with strictly less than $\bar{h}$ total hours worked) and average hours per worker can be used to identify the mean value and dispersion of home productivity, μ_z and σ_z. Intuitively, the persistence of home productivity ρ_z matters for the rate at which workers move between full-time and part-time hours. As for the share of workers bunching at $\bar{h}$ hours, it is directly related to ψ for reasons also discussed in Proposition 3. Note that gender differences in μ_z, ρ_z, σ_z should emerge from this given that the part-time employment share is more than three times higher for female than for male workers, women transition at a twice higher rate between full-time and part-time working hours, and the gender gap in hours per worker is by 5 hours on average.

6. The **vacancy posting cost**. We follow Elsby and Michaels [2013] and calibrate κ so that the expected vacancy posting cost amounts to 14 percent of average quarterly earnings.[23]

7. The **employment separation rate**. We target the rate at which workers separate from employment, i.e., make a transition from employment to nonemployment.[24] In CPS data analyzed in Appendix B, we find that the monthly employment separation rate is 1.79 percent for men vs. 2.63 percent for women, i.e. a 45 percent higher rate. This data moment helps to identify the volatility of shocks to match productivity governed by σ_ε.

8. The **share of exogenous employment separation**. In this section, we have introduced some exogenous separation through δ, in addition to the model's endogenous separations triggered by shocks to match and/or home productivity. Without this ingredient, we need larger shocks to productivity to match the targeted employment separation rate, facing the risk that this could push match productivity (or home productivity, since the underlying parameters are calibrated jointly) into negative territory. To calibrate δ, we assume that exogenous separations account for one quarter (25 percent) of all employment separations. We apply this assumption to both gender groups. The calibrate target for δ pushes σ_ε down towards values similar to those in Bils et al. [2012].

9. The **job-to-job transition rate**. We calculate the share of employed workers who report a change in employer between two consecutive months of CPS interviews. We use it to calibrate on-the-job search intensity, s_e.

10. The **employment rate**. We calculate the average employment rate over the time period covered by our CPS data among workers who have been employed for at least one week in the previous calendar (this information is obtained from the March supplements of the CPS). We add this sample restriction to exclude prime-age individuals who are long

[23] Elsby and Michaels [2013] use empirical evidence based on Silva and Toledo [2009] to calculate this number.

[24] We purposely refrain from calling it the "job separation rate", although this terminology is more common place in the literature. The reason is that there are several types of job separation in addition to transitions from employment to nonemployment: job separations triggered by a job-to-job transition, and transitions from multiple into single jobholding as they entail a separation from either the primary or second job. The term "employment separation rate" is a better descriptor of the moment that we target.

term inactive and who are therefore not well captured by our model. We obtain employment rates of 95.0 and 93.5 percent for men and women, respectively. We use these as calibration targets for the flow cost of working on the first job, ω_1.

11. The **employment share of multiple jobholders**. We target the multiple jobholding share to calibrate the flow cost of working a second job, ω_2. It is important to point out that this is the only targeted data moment that concerns multiple jobholding.

Table 2 presents the outcome of the calibration. The model attributes the difference in working hours between men and women to a combination of differences in the volatility and persistence of productivity in the home sector (σ_z and ρ_z), and to a lesser extent to the mean of home productivity (μ_z). The value of these parameters depends much on that of the Frisch elasticity of labor supply. This is illustrated by Table C1 in the appendix reporting the results of the calibration for $\mathcal{F} = 0.30$ and $\mathcal{F} = 0.60$, and appendix Figure C1 showing the distribution of z for these two calibrations as well as the baseline case ($\mathcal{F} = 0.45$). The other main gender difference in Table 2 lies in the cost of working a second job, ω_2. To account for the fact that women have a similar multiple jobholding share compared to men while being much more likely to work part time, the model sets ω_2 to a higher value for women.

At the same time, the calibration selects similar values of on-the-job search intensity for men and women, namely $s_e = 0.31$. This value is somewhat higher than in on-the-job search models of the kind studied by Jolivet et al. [2006], but not too far off from Mukoyama [2014] who uses $s_e = 0.25$. Notice that in this model, not all on-the-job contacts result in a job-to-job transition: a bad draw of match productivity y_2 would make the worker ignore the outside employer, and for some draws of y_2 the worker becomes a multiple jobholder. In fact, the value of s_e computed in Table 2 is potentially more reliable than many available in the literature because it takes accounts of both job-to-job transitions and multiple jobholding.

4.3 Model fit

The last column of Table 2 shows that the model matches all targeted data moments very well. It does so for both men and women. Table C1 in the appendix shows that the model does equally well when we use a different curvature of the home production function.

Untargeted monthly data moments. Table 3 puts the model to a more stringent test. It compares a set of worker flows generated by the model with their empirical counterparts, all of which are not targeted by the model's calibration.[25]

Panels (a) and (b) of Table 3 show that, on the whole, the model does a good job at explaining worker flows in and out of multiple jobholding. First, it consistently predicts that multiple jobholding is more prevalent among individuals who are working part-time as opposed to full-time on their primary job. That is, the inflow transition rate is about twice as high for part-time workers as for full-time workers. The model slightly understates the transition rate

[25] See Appendix B for additional information on the calculation of data moments reported in Table 3.

Table 1: Externally calibrated parameters

Parameter	Description	Value
β	Monthly discount factor	0.995
$\bar{h}$	Threshold for full-time work	0.4
μ_y	Match prod., uncond. mean	1.0
ρ_y	Match prod., persistence	0.975
α	Tightness elasticity of job filling prob.	0.5
ϕ	Workers' Nash bargaining share	0.5
χ	Matching efficiency	0.50

Notes: The table describes the model parameters that are based on external calibration. The model period is set to be one month.

Table 2: Internally calibrated parameters

Parameter	Description		Value	Targeted moment		Data	Model
γ	Curvature of $g(1-h)$	**M**	0.350	Frisch elasticity of labor supply*	**M**	0.45	0.45
		W	0.281		**W**	0.45	0.45
μ_z	Home prod., uncond. mean	**M**	0.260	Part-time empl. share	**M**	4.92	4.95
		W	0.238		**W**	17.6	17.5
ρ_z	Home prod., persistence	**M**	0.818	Full- to part-time trans. rate	**M**	1.29	1.30
		W	0.938		**W**	3.13	3.16
σ_z	Home prod., standard dev.	**M**	0.121	Average hours per worker	**M**	43.8	43.9
		W	0.087		**W**	38.4	38.2
ψ	Prod. gap at $\bar{h}$ hours	**M**	0.044	Share bunching at full-time hours	**M**	44.2	44.1
		W	0.026		**W**	45.1	44.9
κ	Vacancy posting cost	**M**	0.077	Exp. vacancy cost / qrtly earnings*	**M**	14.0	13.9
		W	0.067		**W**	14.0	14.0
σ_ε	Match prod., standard dev.	**M**	0.061	Empl. separation rate	**M**	1.79	1.80
		W	0.109		**W**	2.63	2.58
δ	Separation shock	**M**	0.005	Share exogenous empl. separation*	**M**	25.0	25.0
		W	0.007		**W**	25.0	25.0
s_e	On-the-job search intensity	**M**	0.314	Job-to-job transition rate	**M**	1.71	1.71
		W	0.311		**W**	1.75	1.77
ω_1	Cost of working job 1	**M**	0.134	Employment rate	**M**	95.0	94.9
		W	0.126		**W**	93.5	93.6
ω_2	Cost of working job 2	**M**	0.043	Multiple jobholding share	**M**	6.57	6.58
		W	0.077		**W**	6.62	6.76

Notes: The table describes the model parameters (left panel) and the model fit to the data moment that is most closely associated to this parameter (right panel). M and W denote model and data moments for respectively men and women. The model period is set to be one month. The data moments except those marked with an asterisk are based data from the Current Population Survey for individuals aged 25 to 54 with some College or higher education; those marked with an asterisk are taken from the literature. All moments except the Frisch elasticity of labor supply and average hours per worker are expressed in percent.

for moving from multiple to single jobholding with a full-time primary job. But it captures the fact that this transition probability is lower than the probability of moving to single jobholding among multiple jobholders with a part-time primary job. Second, the assumption that workers in the model cannot move directly from nonemployment to multiple jobholding is in line with the data. In the reverse direction, the model generates some transitions directly from multiple jobholding to nonemployment, and the order of magnitude is similar to that of the data.

Table 3: Comparison of monthly data and model moments

Untargeted moment		**Data**	**Model**
(a) Multiple jobholding inflow trans. prob.			
Full-time single to multiple jobholding	**M**	2.11	1.82
	W	1.97	1.79
Part-time single to multiple jobholding	**M**	5.79	4.34
	W	4.01	3.90
Nonemployment to multiple jobholding	**M**	0.60	0.00
	W	0.25	0.00
(b) Multiple jobholding outflow trans. prob.			
Full-time multiple to single jobholding	**M**	26.0	22.6
	W	26.4	22.8
Full-time multiple to nonemployment	**M**	0.52	0.15
	W	0.58	0.15
Part-time multiple to single jobholding	**M**	30.1	31.9
	W	31.1	32.6
Part-time multiple to nonemployment	**M**	2.81	0.80
	W	1.55	0.89
(c) Job-to-job trans. prob.			
Job-to-job trans. among single jobholders	**M**	1.65	1.51
	W	1.64	1.49
Job-to-job trans. among multiple jobholders	**M**	2.51	4.41
	W	3.26	5.34

Notes: The table reports a set of model-generated moments and their counterparts based on data from the Current Population Survey for individuals aged 25 to 54 with some College or higher education. M and W denote model and data moments for respectively men and women. All table entries are expressed in percent.

Panel (c) of Table 3 compares job-to-job transition rates respectively for single and multiple jobholders. The model matches the average job-to-job transition rate (Table 2) by slightly underestimating that of single jobholders while overstating that of multiple jobholders, i.e., it generates too many employer changes among workers moving from multiple to single jobholding (Fact 6 in Section 2).[26] Note that this happens *despite* Assumption (A1) which ties a worker to her primary employer. One in four of multiple to single jobholding transitions occur because the first job match is dissolved. The remaining three fourth correspond to two events. In some instances (about 15 percent of the remaining transitions), the worker gives up a second job that, at that point, would not be viable even if it were matched to a single jobholder.[27] But

[26]This difference should not be overly emphasized, however, because the 'same employer' question of the CPS misses a significant portion of all employer-to-employer transitions (Fujita et al. [2023]).

[27]That is, $J_2(y_1, y_2, z)$ (which is proportional to $E(y_1, y_2, z) - E(y_1, z)$ due to Nash bargaining) and $J(y_2, z)$ are both negative. This is likely to occur when y_2 suffers a large negative shock.

in most instances, the worker gives up a second job that would generate a positive surplus if it were matched to a single jobholder, i.e., the 'counterfactual' value $J(y_2, z)$ is positive.

Untargeted yearly data moments. To gain additional understanding, we analyze a set of data moments at the yearly frequency. Our motivation is twofold. First, by looking beyond averages of monthly transition rates, we get a better sense of the degree of *ex post* individual heterogeneity that is generated within the model. Second, as illustrated by the study of Paxson and Sicherman [1996], the PSID provides annual data moments that usefully complement labor market information from the monthly CPS to analyze multiple jobholding.[28]

We first use our model as a laboratory to simulate the trajectory of a large number of multiple jobholders over a long time period and analyze the duration of completed spells of multiple jobholding. This distribution is rightly skewed, with a median duration of completed spells of 2 months and a mean duration of 4.1 months for women and 4.4 months for men. The latter number is different from the average expected duration of multiple jobholding that can be computed out of the hazard rate of transitioning out of multiple jobholding, due to the heterogeneity in transition rates.

Next, we simulate a panel dataset similar to the PSID and calculate yearly labor market outcomes based on those generated in each of the twelve months of simulated data. Panel (a) of Table 4 reports two key statistics: total annual hours worked, and the multiple jobholding share calculated over a 1-year horizon.[29] The model performs well at predicting total annual hours worked. This is perhaps not surprising, given the very good fit with regard to the distribution of weekly hours and to transitions in and out of employment. What is more remarkable is that it also matches the data very well with regard to the annual multiple jobholding share, around 18 percent for men and 21 percent for women. The empirical estimates are in line with Table 1 in Paxson and Sicherman [1996], with a few differences explained by sample selection (ours focuses on workers aged 25 to 54 years old with some College or higher education).[30]

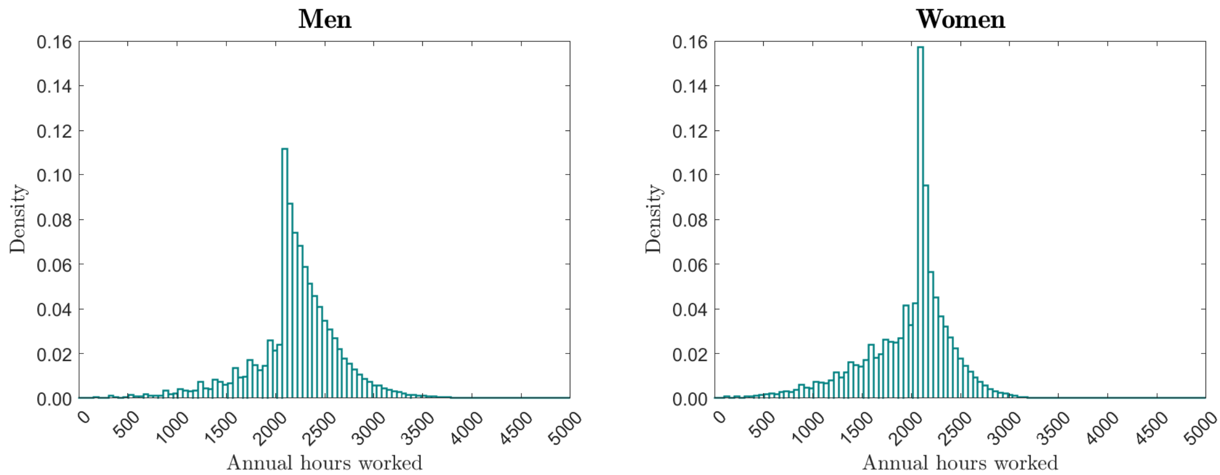
We delve further into the comparison between total annual hours worked in the model and their empirical counterpart from the PSID. The PSID is indeed a primary source of information for analyzing how workers transition within the distribution of annual hours worked; see Chang et al. [2019] for a recent analysis.[31] We begin with the full distribution of these hours as

[28]The March supplements of the CPS are not as useful for that matter. The March CPS records the number of weeks worked in the previous calendar year and the number of *usual* hours per week. There is a lot of heaping at 20 and 40 hours in this variable, which makes it unsuited for the calculation of total annual hours worked. Moreover, the March CPS does *not* record information on multiple jobholding. The survey counts the number of employers (up to three) in the previous calendar year, but it explicitly states that working for more than one employer simultaneously should be counted as only one employer.

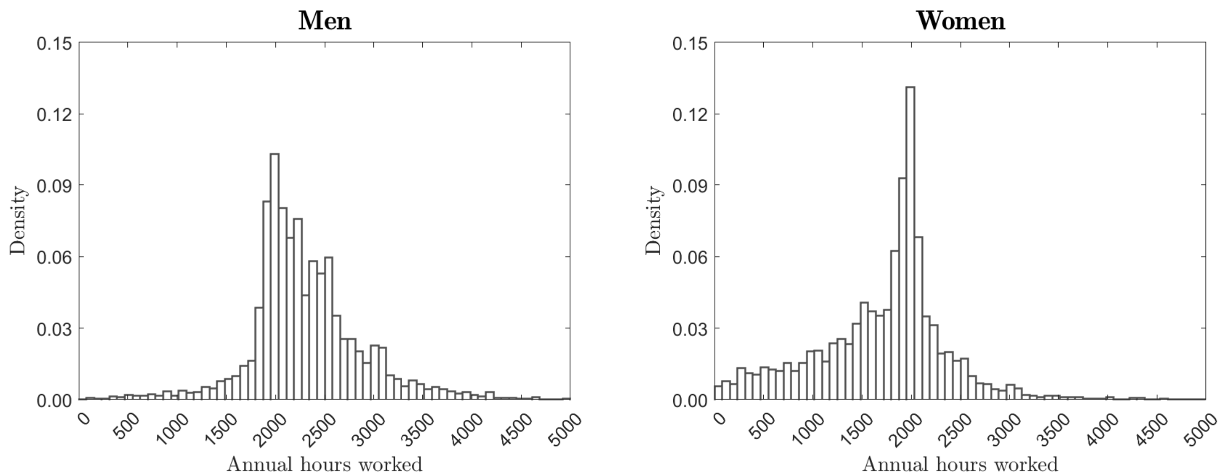
[29]To calculate annual hours, recall that we interpret the model's time endowment as 100 hours per week that can be substituted between the market and production in the home sector.

[30]Table 1 in Paxson and Sicherman [1996] compares PSID and CPS data and shows rates of multiple jobholding that are about 4 times larger in the PSID, as a result of calculating the multiple jobholding share over a 1-year horizon. The discussion surrounding Table 1 of Paxson and Sicherman [1996] clarifies that the PSID is unlikely to confuse job-to-job transitions that happen over the course of the year with multiple jobholding.

[31]We do not conduct a similar analysis for total annual wage earnings since, as pointed out by Paxson and Sicherman [1996], a very large fraction of multiple jobholders in the PSID have missing information on wages from the second job. The authors also note that the issue is worse in the CPS, where second job wage information is missing for five in six multiple jobholders.



(a) Model



(b) Data

Figure 2: Total annual hours worked: Model vs. data

Notes: The panels in this figure plot, for men and women, the distribution of total annual hours worked. Panel (a) is based on simulated data from the baseline model (with parameter values matching a Frisch elasticity of labor supply of 0.45). Panel (b) is based on data from the Panel Study of Income Dynamics for individuals aged 25 to 54 with some College or higher education who report being employed in the previous calendar year.

Table 4: Comparison of yearly data and model moments

Untargeted moment		Data	Model
(a) Hours worked and multiple jobholding			
Total hours worked	**M**	2,260	2,203
	W	1,759	1,879
Multiple jobholding share	**M**	18.4	18.7
	W	20.9	20.7
(b) Changes in hours worked and multiple jobs			
Single jobholding $t-1$ and t	**M**	4.8	5.4
		[452]	[541]
	W	29.1	-6.5
		[514]	[455]
Single jobholding $t-1$; multiple jobholding in t	**M**	244	179
		[673]	[912]
	W	262	125
		[624]	[969]
Multiple jobholding $t-1$; single jobholding in t	**M**	-190	-33.1
		[649]	[821]
	W	-223	-48.5
		[710]	[891]
Multiple jobholding $t-1$ and t	**M**	3.6	9.2
		[547]	[472]
	W	36.6	17.6
		[677]	[393]

Notes: The table reports a set of model-generated moments and their counterparts based on data from the Panel Study of Income Dynamics for individuals aged 25 to 54 with some College or higher education. M and W denote model and data moments for respectively men and women. Multiple jobholding shares are expressed in percent. All other data moments are expressed in hours and correspond to The numbers in brackets are standard deviations.

generated by the model and in the data, respectively in Panels (a) and (b) of Figure 2. As can be seen, in the data for men, there is a fair amount of clustering at around 2,000 hours, and the distribution is skewed to the right of this number. For women, there is also a mass point at 2,000 hours, but there is more mass to the left of the support. The model replicates all these patterns very well.[32]

Panel (b) of Table 4 turns to an important set of facts analyzed in Paxson and Sicherman [1996] concerning the relation between hours worked and multiple jobholding. Similar to Tables 8 and 9 and Figure 1 of their study, we calculate the mean and standard deviation of individual year-to-year changes in annual hours worked, conditional on different transitions (or lack of transition) over two consecutive years. Workers who remain single jobholders experience little change in hours on average. The model replicates this feature, as well as the wide dispersion around the mean observed in the data. It does equally well for workers who are multiple jobholders in two consecutive years. Multiple jobholding is an important channel for adjusting hours upwards. In the data, the average increase is around 250 annual hours. The model generates one half of this number for women, and almost 75 percent of it for men. In the

[32]The model misses somewhat the importance of long hours (over 3,500 annual hours of work) among men. There is a simple 'model fix' that consists in introducing an extra shock to home productivity such that z occasionally jumps to 0, making it optimal to devote all of agents' time endowment to market work. However, this extension does not bring in other insights to the model.

reverse direction, workers give up a second job to reduce their working hours. The model is consistent with this pattern, but it generates a smaller portion of these changes in hours.

5 Determinants of multiple jobholding

In this section, we use the model as a tool to study the factors that affect the multiple jobholding share. We focus on the role of on-the-job search intensity s_e and the flow cost of working a second job ω_2, as these parameters matter for accessing second jobs and for the duration of spells of multiple jobholding.

On-the-job search intensity. Figure 3 reports the effects of changing on-the-job search intensity on the (steady-state equilibrium) share of workers who have a second job. On the horizontal axis, we have normalized the value of s_e in the fully calibrated model to 1 to make the interpretation of the plots straightforward.[33] Figure 3 shows an interesting and perhaps counterintuitive finding: the multiple jobholding share is mostly flat, or is even increasing, when on-the-job search intensity *decreases*. On the one hand, a lower s_e reduces the inflows of multiple jobholding because single jobholders become less likely to meet an outside employer. On the other, conditional on holding a second job, multiple jobholders are less willing to give it up as they anticipate that holding a second job in the future is less likely. As a result, the duration of spells of multiple jobholding lengthens. Given the ambiguous relation between multiple jobholding and s_e, it is quite remarkable that the model comes so close to the multiple jobholding inflows and outflows observed in the data.

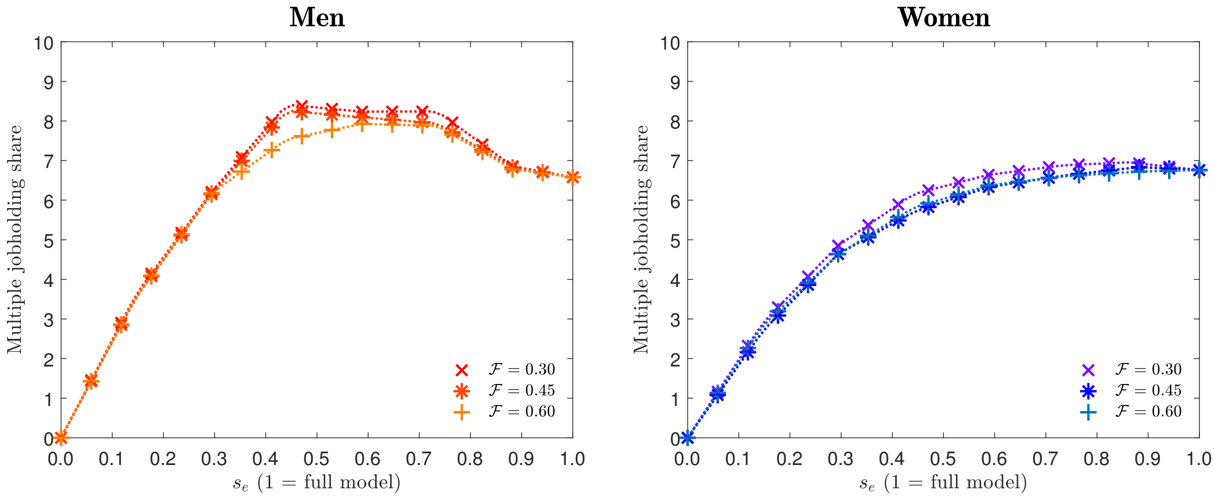


Figure 3: Effects of on-the-job search intensity on multiple jobholding

Notes: The panels in this figure plot, for men and women, the multiple jobholding share computed for different values of on-the-job search intensity, s_e. In the horizontal axes, 1 refers to the full calibrated model. $\mathcal{F}$ denotes the value of the Frisch elasticity of labor supply in the underlying calibration.

[33] The plots show the effects in models calibrated with different targets for the Frisch elasticity, and as a result the models have different parameter values for s_e. As can be seen, the results shown in Figure 3 are very robust to differences of the underlying calibration.

Flow cost of working a second job. The key parameter that allows the model to nail down the multiple jobholding share is ω_2, the flow cost of working a second job.[34] Table 5 compares its value to the average value of monthly earnings, denoted as $\bar{w}$. According to the model, the flow cost of working a second job represents 8 percent of the monthly earnings of men, and 15 percent of those of women. Given existing empirical estimates of the expenditures necessitated by work, this suggests that nonmonetary factors play a role in explaining the flow cost of working a second job.[35]

Table 5: Flow costs of working and the value of leisure

Moment	Description		Value		
			$\mathcal{F} = 0.30$	$\mathcal{F} = 0.45$	$\mathcal{F} = 0.60$
$\frac{\omega_1}{\bar{w}}$	Cost of working job 1 / earnings	**M**	40.7	25.6	20.0
		W	40.9	23.2	16.3
			$\mathcal{F} = 0.30$	$\mathcal{F} = 0.45$	$\mathcal{F} = 0.60$
$\frac{\omega_2}{\bar{w}}$	Cost of working job 2 / earnings	**M**	7.91	8.26	8.42
		W	15.1	14.7	16.7
			$\mathcal{F} = 0.30$	$\mathcal{F} = 0.45$	$\mathcal{F} = 0.60$
$-\frac{\bar{\omega}+\bar{zg}}{\bar{w}}$	Value of leisure / earnings	**M**	79.0	72.9	78.9
		W	78.3	70.3	76.3

Notes: The table reports model-generated moments comparing the flow costs of working and value of leisure to earnings. M and W denote model moments for respectively men and women; $\mathcal{F}$ denotes the value of the Frisch elasticity of labor supply in the underlying calibration. All table entries are expressed in percent.

In Table 5, we also compare the different costs of working to average earnings to contrast them with ω_2 as well as compare our calibration with that of standard search models. Unlike ω_2, the flow cost of working a first job, ω_1, varies with the Frisch elasticity of labor supply, and its value is in the range of 20 to 40 percent of average earnings. This order of magnitude is similar to that of UI benefits in standard job search models. In addition to these flow costs, the model features a value (or utility) of home production through $zg\,(1-h)$. Thus, the total flow value of leisure is the weighted sum of ω_1 and ω_2, denoted as $\bar{\omega}$, and the sum of $zg\,(1-h)$ weighted by the employment distribution of workers, which we denote as $\bar{zg}$. Table 5 shows that the flow value of leisure amounts to between 70 and 80 percent of average earnings, depending on the Frisch elasticity of labor supply. This order of magnitude is similar to the leisure flow value in calibrated version of the DMP model that produce realistic fluctuations of the unemployment rate (Hall and Milgrom [2008], Robin [2011], Fujita and Ramey [2012]).

6 Aggregate effects of multiple jobholding

We now turn to the main quantitative applications of the proposed theory. We analyze the implications of multiple jobholding for the equilibrium outcomes of search models, and for the

[34] We find a larger-than-unity elasticity of the multiple jobholding share with respect to ω_2. This finding resonates with the observation that different MSAs, which are likely characterized by different ω_2's, exhibit vastly different employment shares of multiple jobholders. As pointed out by [Hirsch et al., 2017, p.27], "*Differences in industry and occupation structure, commute times, job churn rates, and ancestry patterns explain a significant share of the [multiple jobholding share] variation across MSAs*".

[35] According to Aguiar and Hurst [2013], work-related expenses represent 5 percent of consumer spending.

inference that these models provide with regard to labor market dynamics.

6.1 Comparison with a standard search model

A key property of our model is that it nests a simpler on-the-job search model in which employed workers cannot have more than one job – the standard setup of all search models. Furthermore, the model calibration is such that none of the targeted moments depend on the number of jobs that workers have, except for the multiple jobholding share. That is, in Table 2, all data moments but the last one can be computed within the context of the search model that rules out multiple jobholding. We use these properties to analyze this 'nested' search model.

Table 6: Internally calibrated parameters (subset) in the model without multiple jobholding

Parameter	**Description**		**Value** *% change from full model*		
			$\mathcal{F} = 0.30$	$\mathcal{F} = 0.45$	$\mathcal{F} = 0.60$
σ_ε	Match prod., standard dev.	**M**	0.039	0.043	0.045
			-23.1%	*-29.8%*	*-33.2%*
		W	0.071	0.101	0.146
			-6.3%	*-8.2%*	*-9.9%*
			$\mathcal{F} = 0.30$	$\mathcal{F} = 0.45$	$\mathcal{F} = 0.60$
s_e	On-the-job search intensity	**M**	0.312	0.314	0.318
			+0.9%	*+1.1%*	*+1.2%*
		W	0.345	0.347	0.352
			+10.6%	*+11.6%*	*+13.1%*

Notes: The table describes a subset of the internally calibrated parameters of the model without multiple jobholding and compares (in italicized numbers) their values to the full model with multiple jobholding. M and W denote model moments for respectively men and women; $\mathcal{F}$ denotes the value of the Frisch elasticity of labor supply in the underlying calibration.

Table 6 summarizes the main differences between the calibrated parameters of the full model vs. the model without multiple jobholding; see Table C2 in the appendix for details. First, shocks to match productivity are substantially less volatile in the model that abstracts from multiple jobholding. Depending on the Frisch elasticity of labor supply, the standard deviation of these shocks, σ_ε, is 23 to 33 percent lower in the calibration for male workers. For female workers, the order of magnitude is lower, with standard deviations that are 6 to 10 percent lower compared to the full model. Foreshadowing the discussion below, these differences reflect the fact that multiple jobholding lowers the employment separation rate. Second, the value of on-the-job search intensity is higher in the model without multiple jobholding, but not by a large margin. For men, the order of magnitude of the difference is only 1 percent, while for women it is 11-13 percent higher.

We then start off from the calibrated model without multiple jobholding (Tables 6 and C2) and introduce the option of holding a second job for $\omega_2 = 0.1$, $\omega_2 = 0.2$, etc. This allows us to study the steady state equilibrium outcomes that are associated with different ω_2's, and hence with different levels of the multiple jobholding share.

Table 7 presents the results.[36] The main takeaway is that multiple jobholding has a non-

[36] In Table 7, we use the model with a Frisch elasticity of labor supply of 0.45. The results are qualitatively similar in the models calibrated to match $\mathcal{F} = 0.30$ or $\mathcal{F} = 0.60$.

Table 7: Effects of multiple jobholding on aggregate labor market outcomes

Moment		**Value** *% change from ref.*			
Multiple jobholding share		0.00	2.50	5.00	7.50
Employment rate	**M**	95.0	95.2	95.4	95.6
		ref.	*+0.23%*	*+0.39%*	*+0.60%*
	W	93.6	93.8	94.1	94.3
		ref.	*+0.30%*	*+0.53%*	*+0.77%*
Tightness	**M**	0.50	0.49	0.49	0.48
		ref.	*-1.17%*	*-2.36%*	*-3.42%*
	W	0.67	0.66	0.65	0.65
		ref.	*-1.84%*	*-2.88%*	*-3.05%*
Empl. separation rate	**M**	1.78	1.69	1.62	1.55
		ref.	*-5.40%*	*-9.18%*	*-13.4%*
	W	2.63	2.48	2.37	2.27
		ref.	*-5.66%*	*-9.95%*	*-13.6%*
Job-to-job transition rate	**M**	1.73	1.74	1.75	1.76
		ref.	*+0.15%*	*+0.93%*	*-1.71%*
	W	1.76	1.81	1.83	1.85
		ref.	*+2.74%*	*+4.02%*	*+5.27%*
Average hours per worker	**M**	44.2	44.26	44.28	44.30
		ref.	*+0.04%*	*+0.10%*	*+0.12%*
	W	39.7	39.8	39.8	39.9
		ref.	*+0.10%*	*+0.22%*	*+0.35%*

Notes: The table reports moments from the model without multiple jobholding computed for values of the flow cost of working a second job, ω_2, that yield a multiple jobholding share of respectively 0%, 2.5%, 5%, and 7.5%. The italicized numbers compare values to those of the model without multiple jobholding (denoted as 'ref.'). M and W denote model moments for respectively men and women. The Frisch elasticity of labor supply in the underlying calibration is 0.45. All table entries except tightness and average hours per worker are expressed in percent.

negligible impact on the extensive margin (the number of workers employed), while it has little effect on the intensive margin (hours per worker). Foremost, introducing multiple jobholding lowers the rate of employment separation: it becomes almost 15 percent lower when multiple jobholders account for a non-trivial share (7.5 percent) of employment. The reason for this is that multiple jobholding makes both the worker and the primary employer better off for a given match y_1 and home productivity z, resulting in fewer destructions of primary job matches. As a result, the employment rate increases. The order of magnitude is plausibly small, i.e., less than a 1 p.p. increase in aggregate employment. The employment gains would be larger if market tightness would remain constant, but as Table 7 shows, multiple jobholding has a negative impact on this variable. Further discussion of this relationship is provided in Subsection 6.2 below. The last rows of Table 7 show that the effects of multiple jobholding on average hours per worker are small. It should be noted that this result is partly driven by a composition effect. Average hours worked in primary jobs decrease (since those jobs are kept alive for lower values of match productivity), while at the same time second jobs bring in additional hours per worker.

We now piece together the results from Tables 6 and 7. First, multiple jobholding lowers the employment separation rate; absent this mechanism, the standard search model underestimates the volatility of match productivity shocks σ_ε consistent with the rate of employment separations observed in the data. Second, multiple jobholding has two effects on the job-to-job transition rate. On the one hand, it lowers the rate because a fraction of on-the-job meetings lead to a spell of multiple jobholding as opposed to a job-to-job transition. On the other, it pushes up the job-to-job transition rate since, as we noted in Subsection 4.3, multiple jobholding introduces a new channel of job-to-job transitions, when the first job match is dissolved and the worker remains employed at the secondary employer. This second channel dominates, so that the full model with multiple jobholding matches the empirical job-to-job rate with a lower value of s_e. Vice versa, the standard search model requires a slightly higher s_e to match the data. The quantitative differences are important for σ_ε and less so for s_e.

6.2 Implications for job creation

To understand further the importance of multiple jobholding for search models, we analyze how it affects the inference that these models provide with regard to labor market dynamics. Specifically, we ask how multiple jobholding affects job creation through the cost of posting vacancies, "the lynchpin of a matching model" (Ljungqvist and Sargent [2007]). We address this question through an illustrative exercise in which we study the effects of an exogenous shift in the on-the-job search intensity parameter, s_e. Two obvious motivations for this exercise is that the job-to-job transition rate contracts markedly during recessions (e.g., Shimer [2005]), and that job-to-job transitions have declined over time, at least relative to the mid-1990s (see Mukoyama [2014] and Fujita et al. [2023]). We confine ourselves to a steady-state analysis of the effects of s_e.

From a search model perspective, a higher s_e increases job creation. This effect is present in the models analyzed here, but is substantially more muted in the full model which takes account of multiple jobholding. To understand why, we examine the job creation condition through the lens of a simple decomposition. Let $\boldsymbol{\Omega} = \{p(y_1,z), d(y_1,y_2,z), \ell(y_1,y_2,z), s_e, \varphi_0(z), \varphi_1(y_1,z)\}$ and $\boldsymbol{S} = \{S(y_1,z), S(y_1,y_2,z)\}$, and use these notations to define the expected surplus conditional on meeting, $\mathbb{E}(\boldsymbol{S}|\boldsymbol{\Omega})$, given by:

$$\begin{aligned}\mathbb{E}(\boldsymbol{S}|\boldsymbol{\Omega}) = &\int\int p(y_1',z')\,S(y_1',z')\,dF_0(y_1')\,dG(z'|z)\,\frac{\varphi_0(z)}{\bar{\varphi}_0+s_e\bar{\varphi}_1}dz \\ &+\int\int\int \big(\ell(y_1',y_2',z')\,p(y_2',z')\,S(y_2',z') + (1-\ell(y_1',y_2',z'))\,p(y_1',z') \\ &\times d(y_1',y_2',z')\,S(y_1',y_2',z')\big)\,dF_0(y_2')\,dF(y_1'|y_1)\,dG(z'|z)\,\frac{s_e\varphi_1(y_1,z)}{\bar{\varphi}_0+s_e\bar{\varphi}_1}dy_1dz. \qquad (34)\end{aligned}$$

Let v denote vacancies, and denote variables from the new equilibrium, viz. the equilibrium with a different on-the-job search intensity, with an upper tilde $(\tilde{.})$. We can decompose the change in vacancies from the baseline to the new equilibrium using the following relation:

$$\tilde{v} - v = \underbrace{\left(\left(\tilde{\bar{\varphi}}_0 + \tilde{s}_e \tilde{\bar{\varphi}}_1\right) - \left(\bar{\varphi}_0 + s_e \bar{\varphi}_1\right)\right)\theta}_{\substack{\text{meeting probability} \\ \text{(extensive search margin)}}}$$

$$+ \underbrace{\left(\frac{\chi}{\kappa}\beta\left(1-\phi\right)\right)^{\frac{1}{\alpha}} \left(\mathbb{E}\left(\boldsymbol{S}|\tilde{\boldsymbol{\Omega}}\right)^{\frac{1}{\alpha}} - \mathbb{E}\left(\boldsymbol{S}|\boldsymbol{\Omega}\right)^{\frac{1}{\alpha}}\right)\left(\tilde{\bar{\varphi}}_0 + \tilde{s}_e \tilde{\bar{\varphi}}_1\right)}_{\substack{\text{matching} \mid \text{meeting} \\ \text{(intensive search margin)}}}$$

$$+ \underbrace{\left(\frac{\chi}{\kappa}\beta\left(1-\phi\right)\right)^{\frac{1}{\alpha}} \left(\mathbb{E}\left(\tilde{\boldsymbol{S}}|\tilde{\boldsymbol{\Omega}}\right)^{\frac{1}{\alpha}} - \mathbb{E}\left(\boldsymbol{S}|\tilde{\boldsymbol{\Omega}}\right)^{\frac{1}{\alpha}}\right)\left(\tilde{\bar{\varphi}}_0 + \tilde{s}_e \tilde{\bar{\varphi}}_1\right)}_{\substack{\text{surplus} \mid \text{matching} \\ \text{(joint surplus sharing)}}}. \quad (35)$$

Consider the effects of raising s_e. The first term in (35) is the effect on the extensive margin of search. The number of job seekers increases, in effective search units, with s_e. Through the matching function, this creates a positive externality on the decision of firms to open more vacancies (since the probability of meeting a job seeker increases). The second term measures the effect of the intensive margin of search. On meeting a job seeker, there is a higher chance that this worker has a job, and hence a lower chance of matching conditional on meeting. This margin contributes negatively to vacancy creation. Third, the surplus of the worker is higher when on-the-job search intensity increases. Since the joint surplus is shared, this third force has a positive impact on incentives for vacancy creation.

Table 8: Elasticity of job creation with respect to on-the-job search intensity

Description		**Multiple jobholding: with**	**without**
Effect on meeting prob.	**M**	+0.78	+0.80
	W	+0.75	+0.78
Effect on matching | meeting	**M**	-1.57	-1.25
	W	-1.70	-1.46
Effect on surplus | matching	**M**	+1.13	+0.88
	W	+1.20	+1.02
Total effect	**M**	+0.34	+0.44
	W	+0.24	+0.33

Notes: The table reports the elasticity of job creation (the number of posted vacancies) with respect to on-the-job search intensity through three channels: probability of meeting, probability of matching conditional on meeting, and expected surplus conditional on matching; as well as their combined effects. M and W denote model moments for respectively men and women. The Frisch elasticity of labor supply in the underlying calibration is 0.45. All table entries are expressed in percent.

We use (35) to calculate the elasticity of job creation with respect to on-the-job search intensity. Table 8 reports the results. In response to a 1-percent increase in s_e, the number of posted vacancies rises by 0.7–0.8 percent through the extensive search margin; higher joint surplus of employment adds a boost of 1.1–1.2 percent; and the intensive margin of search counteracts a substantial fraction of these effects by decreasing vacancies by 1.6–1.7 percent. Combined, these effects imply a job creation elasticity in the range of 0.24-0.34.

The precise value of the elasticity is not on its own the focus of our interest. The main point is rather that the model without multiple jobholding predicts job creation elasticities that are about a third larger, in the 0.33-0.44 range. Table 8 shows that the main driver of this difference is the intensive margin of search, whose effect is amplified by multiple jobholding. In the full model, when an employer meets an employed worker, there is the additional risk that the worker would turn her into a secondary employer. Since for most y_2 and z we have $J(y_2, z) > J_2(y_1, y_2, z)$ (≥ 0), firms are better off if they can avoid matching with these workers, which leads to a more negative effect of the intensive margin of search. In turn, it implies that the standard search model typically overstates the impact of on-the-job search on the incentives for vacancy creation.

7 Conclusion

We develop a search-theoretic model of multiple jobholding. On receiving an on-the-job offer, a worker either moves to the new employer right away, or combines the new job with her job and holds on to her current employer. In exchange for her commitment to the current job, the worker gains a stronger outside option to bargain with the outside employer. This asymmetry between the two employers puts discipline on the rates at which workers transition in and out of second jobs, which can be used to test the model against data on multiple jobholding flows. We show that the model performs well on this test. Furthermore, since the model is general equilibrium, it can be used for counterfactual analysis.

We use the model to study the implications of multiple jobholding for the equilibrium outcomes of search models and for the inference that these models provide regarding labor market dynamics. If workers were allowed to take on second jobs within an otherwise standard search model, the main jobs would survive longer, and as a result, the rate of worker separation from employment would be lower. The flip side is that the standard model underestimates the volatility of shocks to match productivity that trigger separations from employment. It also requires a higher on-the-job search intensity than that in a full model with multiple jobholding since second jobs create an additional channel of job-to-job transitions. Last, since multiple jobholders have a stronger outside option to bargain with the outside employer compared with workers without jobs, multiple jobholding dampens the positive feedback from higher on-the-job search intensity onto additional job creation. The capacity of on-the-job search to amplify labor market fluctuations is therefore lower than suggested by a standard search model.

Our model offers many possibilities for future work. Two of these are especially important. First, there is a perennial policy debate on the design of standard workweek hours and overtime provisions and how these could be used to increase employment while permitting more time for leisure and home production. Given the rich interplay between hours worked, multiple jobholding, and extensive margin adjustments, our model is a natural tool to investigate the impact of these policies quantitatively. Second, empirical research that looks at the effects of income taxation on hours worked finds very large differences in labor supply elasticities when measured using the primary or secondary job. The underlying reasons, which often remain unclear, hold different implications for tax policies, particularly regarding whether second jobs should be exempted from taxes. Our model is suitable for investigating these effects in greater detail. We leave these and other applications for future research.

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Appendices

A Model appendix

Appendices A.1 and A.2 contain the proofs of Propositions 1 and 2, respectively. Appendix A.3 presents the Bellman equations associated to $N(z)$, $E(y_1, z)$, $E(y_1, y_2, z)$, $J(y_1, z)$, $J_1(y_1, y_2, z)$, $J_2(y_1, y_2, z)$ and shows how to combine them with the policy functions from Proposition 2 and the surplus-sharing Equations (10) and (11) to arrive at Equations (19), (22) and (24) of the main text. Appendix A.4 contains the proofs of Proposition A.4.

A.1 Proof of Proposition 1

We derive the results for wages and hours in primary jobs; the steps of the proofs are analogous for second jobs. Since the marginal utility of market consumption equals the marginal cost of paying wages, the first-order condition of the Nash bargaining problem is

$$\phi \frac{1}{E(y_1, z) - N(z)} = (1 - \phi) \frac{1}{J(y_1, z)}.$$

Rearrange this equation and use $S(y_1, z) = J(y_1, z) + E(y_1, z) - N(z)$ to obtain Equation (10). If in addition $f(.)$ and $g(.)$ are differentiable, then the first-order condition for hours is

$$\phi \frac{zg'(1 - h(y_1, z))}{E(y_1, z) - N(z)} = (1 - \phi) \frac{yf'(h(y_1, z))}{J(y_1, z)}.$$

Together with the above first-order condition for wages, this yields $zg'(1 - h(y_1, z)) = yf'(h(y_1, z))$, i.e., Equation (12) of the Proposition.

A.2 Proof of Proposition 2

The first two policy functions, $p(y_1, z)$ and $d(y_1, y_2, z)$, are trivially related to joint match surpluses. We have

$$p(y_1, z) = \mathbb{1}\{J(y_1, z) \geq 0\} = \mathbb{1}\{(1 - \phi) S(y_1, z) \geq 0\} = \mathbb{1}\{S(y_1, z) \geq 0\}$$

and

$$d(y_1, y_2, z) = \mathbb{1}\{E(y_1, y_2, z) \geq E(y_1, z)\} = \mathbb{1}\{\phi S(y_1, y_2, z) \geq 0\} = \mathbb{1}\{S(y_1, y_2, z) \geq 0\}.$$

Next, we look at

$$\begin{aligned}\ell(y_1, y_2, z) = \mathbb{1}\{\max\{E(y_2, z), N(z)\} \geq \max\{&E(y_1, z), E(y_1, z) \\ &+ p(y_1, z)(E(y_1, y_2, z) - E(y_1, z)), N(z)\}\}.\end{aligned}$$

Subtracting $N(z)$ on both side of the inequality yields

$$\begin{aligned}\ell(y_1, y_2, z) = \mathbb{1}\{\max\{E(y_2, z) - N(z), 0\} \geq \max\{&E(y_1, z) - N(z), E(y_1, z) \\ &+ p(y_1, z)(E(y_1, y_2, z) - E(y_1, z)) - N(z), 0\}\}.\end{aligned}$$

On the one hand, we have

$$\max\{E(y_2,z)-N(z),0\}=\max\{\phi S(y_2,z),0\}=p(y_2,z)\,\phi S(y_2,z).$$

On the other, $E(y_1,z)-N(z)=\phi S(y_1,z)$ and $E(y_1,y_2,z)-E(y_1,z)=\phi S(y_1,y_2,z)$, and thus we have

$$\begin{aligned}
&\max\{E(y_1,z)-N(z),E(y_1,z)+p(y_1,z)(E(y_1,y_2,z)-E(y_1,z))-N(z),0\}\\
&\quad=\max\{\phi S(y_1,z),\phi(S(y_1,z)+p(y_1,z)\,S(y_1,y_2,z)),0\}\\
&\quad=\max\{\max\{\phi S(y_1,z),\phi(S(y_1,z)+p(y_1,z)\,S(y_1,y_2,z))\},0\}\\
&\quad=\max\{\phi S(y_1,z)+\max\{0,p(y_1,z)\,\phi S(y_1,y_2,z)\},0\}\\
&\quad=\max\{\phi S(y_1,z)+p(y_1,z)\,d(y_1,y_2,z)\,\phi S(y_1,y_2,z),0\}.
\end{aligned}$$

If $S(y_1,z)\geq 0$ then $S(y_1,z)+p(y_1,z)\,d(y_1,y_2,z)\,S(y_1,y_2,z)\geq 0$, so that we also have

$$\begin{aligned}
\max\{\phi S(y_1,z)+p(y_1,z)\,d(y_1,y_2,z)\,\phi S(y_1,y_2,z),0\}=p(y_1,z)\,\phi\,(S(y_1,z)&\\
+\,d(y_1,y_2,z)\,S(y_1,y_2,z))&
\end{aligned}$$

and we arrive at

$$\ell(y_1,y_2,z)=\mathbb{1}\{p(y_2,z)\,S(y_2,z)\geq p(y_1,z)\,(S(y_1,z)+d(y_1,y_2,z)\,S(y_1,y_2,z))\}.$$

A.3 Bellman equations

The asset value of a non-employed worker is

$$\begin{aligned}
N(z)&=\beta\int\left((1-\lambda_0)\,N(z')+\lambda_0\int\max\{E(y_1',z'),N(z')\}\,dF_0(y_1')\right)dG(z'|z)\\
&=\beta\int\left(N(z')+\lambda_0\int\max\{E(y_1',z')-N(z'),0\}\,dF_0(y_1')\right)dG(z'|z)
\end{aligned}$$

and since $E(y_1,z)-N(z)=\phi S(y_1,z)$, we have

$$\begin{aligned}
N(z)&=\beta\int\left(N(z')+\lambda_0\int\max\{\phi S(y_1',z'),0\}\,dF_0(y_1')\right)dG(z'|z)\\
&=\beta\int\left(N(z')+\lambda_0\phi\int p(y_1',z')\,S(y_1',z')\,dF_0(y_1')\right)dG(z'|z)
\end{aligned}$$

using the policy function $p(y_1,z)=\mathbb{1}\{S(y_1,z)\geq 0\}$.

The asset value of a single jobholder, $\tilde{E}(w,h;y_1,z)$, is

$$\begin{aligned}
\tilde{E}(w,h;y_1,z)=w-\omega_1+zg(1-h)+\beta\int\Bigg((1-\lambda_1)\int\max\{E(y_1',z'),&\\
N(z')\}\,dF(y_1'|y_1)+\lambda_1\int\int\max\{E(y_1',z')+p(y_1',z')\,(E(y_1',y_2',z')-E(y_1',z'))&,\\
E(y_1',z'),E(y_2',z'),N(z')\}\,dF_0(y_2')\,dF(y_1'|y_1)\Bigg)\,dG(z'|z)&
\end{aligned}$$

$$= w - \omega_1 + zg(1-h) + \beta \int \Big(N(z') + (1-\lambda_1) \int \max\{E(y_1', z')$$

$$- N(z'), 0\} dF(y_1'|y_1) + \lambda_1 \int\int \max\{E(y_1', z') + p(y_1', z')(E(y_1', y_2', z') - E(y_1', z'))$$

$$-N(z'), E(y_1', z') - N(z'), E(y_2', z') - N(z'), 0\} dF_0(y_2') dF(y_1'|y_1) \Big) dG(z'|z).$$

Since $E(y_1, z) - N(z) = \phi S(y_1, z)$ and $E(y_1, y_2, z) - E(y_1, z) = \phi S(y_1, y_2, z)$ via the surplus-sharing Equations (10) and (11), it follows that

$$\tilde{E}(w, h; y_1, z) = w - \omega_1 + zg(1-h) + \beta \int \Big(N(z') + (1-\lambda_1)$$

$$\times \int \max\{\phi S(y_1', z'), 0\} dF(y_1'|y_1) + \lambda_1 \int\int \max\{\phi(S(y_1, z) + p(y_1, z) S(y_1, y_2, z)),$$

$$\phi S(y_1', z'), \phi S(y_2', z'), 0\} dF_0(y_2') dF(y_1'|y_1) \Big) dG(z'|z).$$

Recall that $E(y_1, z) = \tilde{E}(w(y_1, z), h(y_1, z); y_1, z)$ (see (6)). By using the policy functions and results from Appendix A.2 we obtain

$$E(y_1, z) = w(y_1, z) - \omega_1 + zg(1 - h(y_1, z)) + \beta \int \Big(N(z') + (1-\lambda_1) \int \phi p(y_1', z')$$

$$\times S(y_1', z') dF(y_1'|y_1) + \lambda_1 \int\int \phi(\ell(y_1', y_2', z') p(y_2', z') S(y_2', z') + (1 - \ell(y_1', y_2', z'))$$

$$\times p(y_1', z')(S(y_1', z') + d(y_1', y_2', z') S(y_1', y_2', z'))) dF_0(y_2') dF(y_1'|y_1) \Big) dG(z'|z)$$

given that: if $\ell(y_1, y_2, z) = 1$, then the worker receives $\phi S(y_2, z)$ if $p(y_2, z) = 1$; and if $\ell(y_1, y_2, z) = 0$ and $p(y_1, z) = 1$, the worker receives $\phi S(y_1, z)$ and in addition she receives $\phi S(y_1, y_2, z)$ if $d(y_1, y_2, z) = 1$.

The asset value of a multiple jobholder, $\tilde{E}(w, h; y_1, y_2, z)$, is

$$\tilde{E}(w, h; y_1, y_2, z) = w(y_1, z) + w - \omega_1 - \omega_2 + zg(1 - h(y_1, z) - h)$$

$$+ \beta \int \Big(\Big(\int (1 - p(y_1', z')) dF(y_1'|y_1) \Big) \int \max\{E(y_2', z'), N(z')\} dF(y_2'|y_2)$$

$$+ \int\int (p(y_1', z')(E(y_1', z') + d(y_1', y_2', z')(E(y_1', y_2', z')$$

$$-E(y_1', z')))) dF(y_2'|y_2) dF(y_1'|y_1) \Big) dG(z'|z)$$

$$= w(y_1, z) + w - \omega_1 - \omega_2 + zg(1 - h(y_1, z) - h)$$

$$+ \beta \int \Big(N(z') + \Big(\int (1 - p(y_1', z')) dF(y_1'|y_1) \Big) \int \max\{E(y_2', z') - N(z')$$

$$, 0\} dF(y_2'|y_2) + \int\int (p(y_1', z')(E(y_1', z') - N(z') + d(y_1', y_2', z')(E(y_1', y_2', z')$$

$$-E(y_1', z')))) dF(y_2'|y_2) dF(y_1'|y_1) \Big) dG(z'|z)$$

where we have made use directly of the policy functions $p(y_1, z)$ and $d(y_1, y_2, z)$. Surplus sharing, the definition $E(y_1, y_2, z) = \tilde{E}(w(y_1, y_2, z), h(y_1, y_2, z); y_1, y_2, z)$ (see (7)), and making use of $p(y_2, z)$ for the second job yields

$$\begin{aligned}
E(y_1, y_2, z) &= w(y_1, z) + w(y_1, y_2, z) - \omega_1 - \omega_2 + zg(1 - h(y_1, z) - h(y_1, y_2, z)) \\
&\quad + \beta \int \Bigg(N(z') + \left(\int (1 - p(y_1', z'))\, dF(y_1'|y_1) \right) \int \max\{\phi S(y_2', z'), 0\}\, dF(y_2'|y_2) \\
&\qquad + \int \int (p(y_1', z')(\phi S(y_1', z') + d(y_1', y_2', z') \\
&\qquad\qquad \times \phi S(y_1', y_2', z')))\, dF(y_2'|y_2)\, dF(y_1'|y_1) \Bigg) dG(z'|z) \\
&= w(y_1, z) + w(y_1, y_2, z) - \omega_1 - \omega_2 + zg(1 - h(y_1, z) - h(y_1, y_2, z)) \\
&\quad + \beta \int \Bigg(N(z') + \left(\int (1 - p(y_1', z'))\, dF(y_1'|y_1) \right) \int \phi p(y_2', z') S(y_2', z')\, dF(y_2'|y_2) \\
&\quad + \int \int (\phi p(y_1', z')(S(y_1', z') + d(y_1', y_2', z') S(y_1', y_2', z')))\, dF(y_2'|y_2)\, dF(y_1'|y_1) \Bigg) dG(z'|z).
\end{aligned}$$

Next, the asset value of employing a single jobholder, $\tilde{J}(w, h; y_1, z)$, is

$$\begin{aligned}
\tilde{J}(w, h; y_1, z) &= y_1 f(h) - w + \beta \int \Bigg(\lambda_1 \int \int ((1 - \ell(y_1', y_2', z'))\, p(y_1', z') \\
&\quad \times ((1 - d(y_1', y_2', z'))\, J(y_1', z') + d(y_1', y_2', z')\, J_1(y_1', y_2', z')))\, dF_0(y_2')\, dF(y_1'|y_1) \\
&\qquad + (1 - \lambda_1) \int \max\{J(y_1', z'), 0\}\, dF(y_1'|y_1) \Bigg) dG(z'|z) \\
&= y_1 f(h) - w + \beta \int \Bigg(\lambda_1 \int \int ((1 - \ell(y_1', y_2', z'))\, p(y_1', z') \\
&\quad \times (J(y_1', z') + d(y_1', y_2', z')(J_1(y_1', y_2', z') - J(y_1', z'))))\, dF_0(y_2')\, dF(y_1'|y_1) \\
&\qquad + (1 - \lambda_1) \int \max\{J(y_1', z'), 0\}\, dF(y_1'|y_1) \Bigg) dG(z'|z)
\end{aligned}$$

where, again, we have made direct use of the policy functions $p(y_1, z)$ and $d(y_1, y_2, z)$ to simplify notations. Note that $p(y_1, z)$ multiplies $J_1(y_1, y_2, z)$ as participation of the primary employer must be ensured. With the surplus sharing rule and definition of $J(y_1, z)$ (see (6)), we obtain

$$\begin{aligned}
J(y_1, z) &= y_1 f(h(y_1, z)) - w(y_1, z) + \beta \int \Bigg(\lambda_1 \int \int ((1 - \ell(y_1', y_2', z'))\, p(y_1', z') \\
&\quad \times ((1 - \phi) S(y_1', z') + d(y_1', y_2', z')(J_1(y_1', y_2', z') - (1 - \phi) S(y_1', z'))))\, dF_0(y_2')\, dF(y_1'|y_1) \\
&\qquad + (1 - \lambda_1) \int p(y_1', z')(1 - \phi) S(y_1', z')\, dF(y_1'|y_1) \Bigg) dG(z'|z).
\end{aligned}$$

In order to write the asset value of the primary employer, recall that the value in the continuation period depends on $d(y_1, y_2, z)$, the worker's decision to keep the second job, and on the constraint that the job remains viable captured by $p(y_1, z)$. Thus, the asset value of the primary employer is

$$\begin{aligned}
J_1(y_1, y_2, z) &= y_1 f(h(y_1, z)) - w(y_1, z) + \beta \int \int \int (d(y_1', y_2', z')\, p(y_1', z')\, J_1(y_1, y_2, z) \\
&\quad + (1 - d(y_1', y_2', z')) \max\{J(y_1', z'), 0\})\, dF(y_2'|y_2)\, dF(y_1'|y_1)\, dG(z'|z)
\end{aligned}$$

$$= y_1 f(h(y_1,z)) - w(y_1,z) + \beta \int\int p(y_1',z') \Bigg((1-\phi) S(y_1',z') + \int \big(d(y_1',y_2',z')$$
$$\times (J_1(y_1',y_2',z') - (1-\phi) S(y_1',z')) dF(y_2'|y_2) \Bigg) dF(y_1'|y_1) \, dG(z'|z).$$

The last equation uses the surplus sharing rule, and so we arrive at equation (24).

Lastly, the asset value for the secondary employer of a multiple jobholder, $\tilde{J}_2(y_1,y_2,z)$, is

$$\tilde{J}_2(w,h;y_1,y_2,z) = y_2 f(h) - w + \beta \int \Bigg(\int\int \big(d(y_1',y_2',z')\, p(y_1',z')$$
$$\times \max\{J_2(y_1',y_2',z'),0\}\big) dF(y_2'|y_2)\, dF(y_1'|y_1) + \left(\int (1-p(y_1',z'))\, dF(y_1'|y_1)\right)$$
$$\times \left(\int \max\{J(y_2',z'),0\}\, dF(y_2'|y_2)\right)\Bigg) dG(z'|z),$$

taking account of the workers' commitment $p(y_1,z) = \mathbb{1}\{J(y_1',z') \geq 0\}$ towards her primary employer. Plugging in the outcomes of Nash bargaining $w(y_1,y_2,z)$ and $h(y_1,y_2,z)$, and by definition that $J_2(y_1,y_2,z) = \tilde{J}_2(w(y_1,y_2,z), h(y_1,y_2,z); y_1,y_2,z)$ (see (7)), we arrive at

$$J_2(y_1,y_2,z) = y_2 f(h(y_1,y_2,z)) - w(y_1,y_2,z) + \beta \int \Bigg(\int\int \big(d(y_1',y_2',z')\, p(y_1',z')$$
$$\times \max\{(1-\phi) S(y_1',y_2',z'),0\}\big) dF(y_2'|y_2)\, dF(y_1'|y_1) + \left(\int (1-p(y_1',z'))\, dF(y_1'|y_1)\right)$$
$$\times \left(\int \max\{(1-\phi) S(y_2',z'),0\}\, dF(y_2'|y_2)\right)\Bigg) dG(z'|z)$$
$$= y_2 f(h(y_1,y_2,z)) - w(y_1,y_2,z) + \beta \int \Bigg(\int\int \big(d(y_1',y_2',z')\, p(y_1',z')$$
$$\times (1-\phi) S(y_1',y_2',z')\, dF(y_2'|y_2)\, dF(y_1'|y_1) + \left(\int (1-p(y_1',z'))\, dF(y_1'|y_1)\right)$$
$$\times \left(\int p(y_1',z')(1-\phi) S(y_2',z')\, dF(y_2'|y_2)\right)\Bigg) dG(z'|z).$$

Here, we have used the policy functions from Proposition 2 and the surplus sharing equations. In particular, observe that $d(y_1',y_2',z') \max\{J_2(y_1',y_2',z'),0\} = d(y_1',y_2',z')\, J_2(y_1',y_2',z')$ since $d(y_1',y_2',z') = \mathbb{1}\{S(y_1',y_2',z') \geq 0\} = \mathbb{1}\{J_2(y_1',y_2',z') \geq 0\}$.

To complete the derivation, add up the last equations we have obtained for $E(y_1,z)$ and $J(y_1,z)$ and subtract $N(z)$ in order to arrive at Equation (19) ($S(y_1,z)$). Similarly, add up the last equations we have obtained for $E(y_1,y_2,z)$ and $J_2(y_1,y_2,z)$ and subtract $E(y_1,z) = \phi S(y_1,z) + N(z)$ to arrive at Equation (22) ($S(y_1,y_2,z)$). Finally, to recover the wage functions, rearrange the last equation we have obtained for $J(y_1,z)$ and $J_2(y_1,y_2,z)$ to compute $w(y_1,z)$ and $w(y_1,y_2,z)$. In these calculations, use $J(y_1,z) = (1-\phi) S(y_1,z)$ and $J_2(y_1,y_2,z) = (1-\phi) S(y_1,y_2,z)$ on the left-hand side of each equation.

A.4 Proof of Proposition 3

Since the function $f(.)$ in (30) is not differentiable everywhere, we cannot apply the second part of Proposition 1. However, the first part of the Proposition, i.e., surplus sharing through wages, remains valid. For primary jobs (the logic is analogous for second jobs) we have $E(y_1, z) - N(z) = \phi S(y_1, z)$ and $J(y_1, z) = (1-\phi) S(y_1, z)$. Substituting into the Nash product, we obtain

$$(E(y_1, z) - N(z))^{\phi} J(y_1, z)^{1-\phi} = \phi^{\phi} (1-\phi)^{1-\phi} S(y_1, z).$$

Thus, agents choose hours worked to maximize the joint surplus. Given rebargaining every period, this amounts to maximizing the sum of market and home productions, $yf(h) + zg(1-h)$.

The specific functional forms for $g(1-h)$ and $f(h)$ in respectively (29) and (30) give rise to the following possibilities, illustrated in Figure A1:

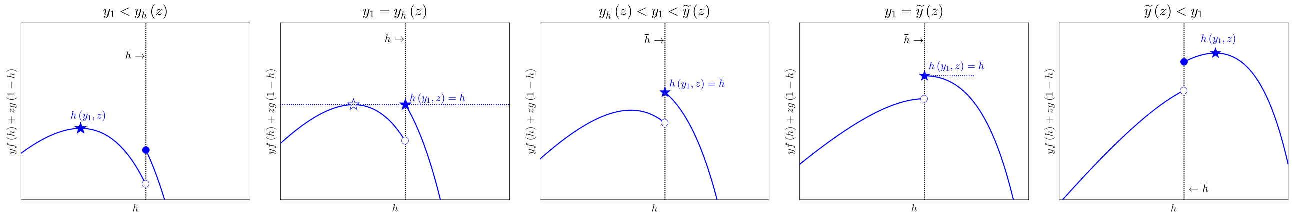

Figure A1: Illustration of Proposition 3

First, there can be an interior maximum of $yf(h) + zg(1-h)$ attained at some h to the left of $\bar{h}$. Using the first-order condition, we have: $h(y_1, z) = 1 - \left(\frac{z}{(1-\psi)y_1}\right)^{\gamma}$. As y increases, $yf(h) + zg(1-h)$ shifts up, and there is a value $y_{\bar{h}}(z)$ such that the interior solution to the left of $\bar{h}$ yields the same maximum value as $yf(\bar{h}) + zg(1-\bar{h})$. Thus, $y_{\bar{h}}(z)$ is defined by

$$y_{\bar{h}}(z) f(h(y_{\bar{h}}(z), z)) + zg(1 - h(y_{\bar{h}}(z), z)) = y_{\bar{h}}(z) f(\bar{h}) + zg(1-\bar{h}), \tag{36}$$

with $h(y_{\bar{h}}(z), z) = 1 - \left(\frac{z}{(1-\psi)y_{\bar{h}}(z)}\right)^{\gamma}$. For values of y_1 greater than $y_{\bar{h}}(z)$ but strictly below some $\widetilde{y}(z)$, there is a local maximum in the $[0, \bar{h})$ interval, but the global maximum is given by $h(y_1, z) = \bar{h}$. At $y_1 = y_{\bar{h}}(z)$, the maximum attained at $h = \bar{h}$ satisfies the first-order condition. As a result, we have:

$$\widetilde{y}(z) = \frac{z\left(1-\bar{h}\right)^{-\frac{1}{\gamma}}}{1-\psi}. \tag{37}$$

For values of y_1 above $\widetilde{y}(z)$, the interior solution to the right of $\bar{h}$ is the global maximum.

We define likewise a cutoff value for second jobs $y_{\bar{h}}(y_1, z)$ which satisfies

$$\begin{aligned} y_{\bar{h}}(y_1, z) f(h(y_1, y_{\bar{h}}(y_1, z), z)) + zg(1 - h(y_1, z) - h(y_1, y_{\bar{h}}(y_1, z), z)) \\ = y_{\bar{h}}(y_1, z) f(\bar{h}) + zg\left(1 - h(y_1, z) - \bar{h}\right), \end{aligned} \tag{38}$$

with $h(y_1, y_{\bar{h}}(y_1, z), z) = 1 - h(y_1, z) - \left(\frac{z}{(1-\psi)y_{\bar{h}}(y_1,z)}\right)^{\gamma}$. Observe that $1 - h(y_1, z)$ is the upper bound on the hours that can be allocated to the second job. The upper threshold $\widetilde{y}(y_1, z)$ that is the analogous of (37) is given by

$$\widetilde{y}(y_1, z) = \frac{z\left(1 - h(y_1, z) - \bar{h}\right)^{-\frac{1}{\gamma}}}{1-\psi}. \tag{39}$$

This completes the proof of Prop. 3 (with Equations (36)–(39) mentioned in the Proposition).

B Data appendix

The main data source of our analysis is the Current Population Survey (CPS). The CPS is a survey of households administered by the U.S. Census Bureau under the auspices of the U.S. Bureau of Labor Statistics. The CPS is a leading source of information on labor market dynamics based on the analysis of worker flows. In addition, it contains earnings information that can be used to further characterize worker transitions across labor market states.

B.1 Transition probabilities

We use a stock-flow framework to measure transitions in and out of multiple jobholding. In each period t, individuals are classified into one of the following states: multiple jobholding with a full-time primary job (F_M) or with a part-time primary job (P_M), single jobholding with a full-time job (F_S) or with a part-time job (P_S), and nonemployment (N). We let the vector $\boldsymbol{s}_t$ contain the number of individuals (stocks) in each of these states:

$$\boldsymbol{s}_t = [\ \underbrace{F_M \quad P_M}_{M} \ \underbrace{F_S \quad P_S}_{S} \quad N\ ]_t',$$

where $M = F_M + P_M$ (resp. $S = F_S + P_S$) is the number of multiple jobholders (resp. single jobholders) in period t. The evolution of $\boldsymbol{s}_t$ is described by means of a discrete-time, first-order Markov chain: $\boldsymbol{s}_t = \boldsymbol{\Pi}_t \boldsymbol{s}_{t-1}$, where $\boldsymbol{\Pi}_t$ is the stochastic matrix of transition probabilities across labor market states i and j. Each of these transition probabilities is measured by the gross flow of workers from state i to state j at time t divided by the stock of workers in state i at time $t-1$.

Before feeding the Markov chain with the stocks and worker flows data, we clear those from systematic seasonal variations. We then perform adjustments to control for margin error discrepancies and time-aggregation bias. The margin error adjustment corrects the transition probabilities of $\boldsymbol{\Pi}_t$ to account for the fact that $\boldsymbol{s}_t$ ignores a number of possible statuses, such as retirement, death, etc. or, more importantly in the context of our analysis, self-employment. The time-aggregation bias correction addresses the fact that the discrete-time (monthly) probabilities miss some of the transitions that occur at a higher frequency. In this context, it increases transition probabilities between M and S at the expense of transitions between M and N, presumably because the latter often involve an intervening spell of single jobholding.

B.2 Same employer, duties and activities

Starting in 1994, the CPS measures whether respondents change employers, duties or activities between two consecutive months of interview. Specifically, re-interviewed respondents are asked the following questions:

Q1. "Last month, it was reported that [name/you] worked for [input company name]. [Do/Does] [you/he/she] still work for [input company name] at [your/his/her] main job?"

Q2. "Have the usual activities and duties of [your/his/her] job changed since last month?"

Q3. "Last month [name/you] [was/were] reported as [a/an] [input occupation] and [your/his/her] usual activities were [input duties 1] [input duties 2]. Is this an accurate description of [your/his/her] current job?"

The first in this series of questions refers explicitly to the respondent's 'main job'. While the term 'main job' may be as perceived by respondents, the CPS glossary defines it as "the one at which they usually work the greatest number of hours." (see https://www.bls.gov/cps/definitions.htm). The other questions clearly refers to what has been identified as the 'main job' through Question Q1.

Table B1: Multiple jobholding: Employer, duties and activities on the main job

		Transition probability	% whose main job has: same employer	same duties	same activities
(a) Multiple jobholding inflow					
Full-time single to multiple jobholding	**M**	2.11	95.3	98.5	98.1
	W	1.97	95.0	98.5	97.9
Part-time single to multiple jobholding	**M**	5.79	85.2	98.4	98.5
	W	4.01	88.5	98.4	98.7
(b) Multiple jobholding outflow					
Full-time multiple to single jobholding	**M**	26.0	96.8	98.7	99.0
	W	26.4	96.0	98.7	98.9
Part-time multiple to single jobholding	**M**	30.1	91.3	98.1	98.7
	W	31.1	89.6	98.8	98.9

Notes: The table reports the monthly transition probabilities between single and multiple jobholding and proportion of workers who, upon switching employment status, report that their main job remains with the same employer, entails the same duties or the same main activities. Data come from the Current Population Survey for individuals aged 25 to 54 with some College or higher education. M and W denote data moments for respectively men and women. All table entries are expressed in percent.

We use the answers to the above questions to analyze transitions towards (i.e., the inflow of) and from (the outflow) multiple jobholding in respectively Panels (a) and (b) of Table B1. For both men and women, transitions from single to multiple jobholding occur more frequently when working part-time on the main job. Most strikingly, we find that less than 2 percent of workers report a change in the duties or activities of the main job. Similar patterns hold true for transitions in the reverse direction. What is more, we find that the vast majority of workers remain with their main employer upon taking on, as well as giving up, a second job. For transitions between part-time single (P_S) to multiple jobholding (M), 15 percent of workers report a change in employer for the main job, which may be because they take on a second job that entails more hours worked. When switching from part-time multiple (P_M) to single jobholding (S), about 10 percent of workers report a change in employer, which shows that workers typically hold on the job associated with more hours worked.

B.3 Wage and hours worked regressions

Here we provide details about the wage and hours worked regressions presented in Figure 1, Section 2. In the CPS, respondents are interviewed for four consecutive months, are rotated out of the survey for eight months, and are included in the survey again for four consecutive months. As shown in Table B2, the labor force status, number of (usual and actual) hours workers, number of jobs held, and primary occupation and industry are recorded in all eight interviews. Information on whether the respondent is at the same employer as in the previous month (Q1 in Subsection B.2) pertains to all interviews except the first (MIS = 1) and fifth (MIS = 5) ones. As for the earnings information about the primary job, it is available for the Outgoing Rotation Groups of the survey, i.e., MIS = 4 and MIS = 8.

To analyze the wage impact of taking on a second job, we select respondents i who are employed and hold only one job in MIS = 4, and who remain employed in MIS = 5 through MIS = 8. $m_{i,t}$ is an indicator that takes the value of one if i takes on a second job at any point

Table B2: Current Population Survey structure

Survey tenure month	**1**	**2**	**3**	**4**	...	**13**	**14**	**15**	**16**
Month in sample (MIS)	1	2	3	4		5	6	7	8
LF status, hours, number of jobs held	✔	✔	✔	✔		✔	✔	✔	✔
Same employer		✔	✔	✔			✔	✔	✔
Occupation and industry	✔	✔	✔	✔		✔	✔	✔	✔
Earnings				✔					✔

Notes: The table describes the structure of the monthly Current Population Survey. A checkmark indicates that the the information is included in the corresponding survey tenure month of CPS respondents.

in MIS 5 through 8, and is zero otherwise. To minimize effects of other sources of changes in the dependent variable, we focus on respondents who report that they remain employed at the same employer in MIS 6, 7 and 8. These samples are denoted as "Same employer" in Figure 1. Since ideally we would like to control for changes in employers during the 8-month gap that takes place between survey tenure months 4 and 13, we also consider samples where both the primary occupation and industry of employment remain the same in MIS = 4 and MIS = 8.[37] Finally, we implement the "same employer", "same occupation" and "same industry" sample restrictions together to run the estimation.

For the data samples used in the analysis of transitions out of multiple jobholding, we select respondents i who are employed and hold more than one job in MIS = 4 and who remain employed through MIS = 8. In a way symmetric to our analysis of the inflows, we define $m_{i,t}$ as the indicator for returning to single jobholding at any point after MIS = 4 to analyze the outflows.

Panel (a) of Figure 1 in Section 2 reports the results from 36 linear regressions that are variants of: $\triangle \log w_{i,t} = \alpha + \beta m_{i,t} + \mathbf{X}'_{\mathbf{i,t}}\gamma + \varepsilon_{i,t}$ for the two definitions of $m_{i,t}$, and where the regressions are run on different samples and may include state and time fixed effects.[38,39] In all instances, the regressions fail to detect any statistically significant impact of having a second job on the hourly wage of the primary job. In Panel (b) of Figure 1, we consider an analogous set of regressions, with changes in (the log of) hours worked as the left-hand side variable. Hours worked of the primary job remain unchanged upon taking a second job. Hours worked seem to increase on dropping a second job, but the effect becomes becomes statistically insignificant once we account for state-time fixed effects. We conclude that the effects for hours worked are essentially similar to those on wages, i.e., zero impact.

C Additional results

Table C1 is the analogue of Table 2 in the main text. The table presents the outcomes of different calibrations that target values of the Frisch elasticity of 0.30 and 0.60. The other calibration targets are the same as in the baseline calibration. The model fit as measured in the last set of columns of Table C1 remains very similar.

[37]We use classifications provided by IPUMS-CPS (https://cps.ipums.org/cps/) to harmonize the occupational and industry classification available in the monthly CPS data. For both classifications, we consider the 3-digit levels, which include 450 occupations and 234 industries.

[38]We exclude respondents whose hourly wage in either MIS = 4 or MIS = 8 is lower than \$2.50 or greater than \$500 per hour. For the regressions with hours as the dependent variable, we exclude those with usual hour worked lower than 1 or greater than 80 hours per week in either MIS = 4 or MIS = 8.

[39]There are 264 dummies for the time fixed effects, and 13,464 dummies for the state $\times$ time fixed effects.

Table C1: Internally calibrated parameters: Lower ($\mathcal{F} = 0.30$) and higher ($\mathcal{F} = 0.60$) Frisch elasticity of labor supply

Parameter	Description		Value		Targeted moment		Data		Model	
			$\mathcal{F} = 0.30$	$\mathcal{F} = 0.60$			$\mathcal{F} = 0.30$	$\mathcal{F} = 0.60$	$\mathcal{F} = 0.30$	$\mathcal{F} = 0.60$
γ	Curvature of $g(1-h)$	**M**	0.234	0.467	Frisch elasticity of labor supply*	**M**	0.30	0.60	0.30	0.60
		W	0.187	0.374		**W**	0.30	0.60	0.30	0.60
μ_z	Home prod., uncond. mean	**M**	0.124	0.374	Part-time empl. share	**M**	4.92		4.93	4.87
		W	0.093	0.378		**W**	17.6		17.9	17.8
ρ_z	Home prod., persistence	**M**	0.799	0.833	Full- to part-time trans. prob.	**M**	1.29		1.27	1.18
		W	0.982	0.995		**W**	3.13		3.25	3.39
σ_z	Home prod., standard dev.	**M**	0.066	0.146	Average hours per worker	**M**	43.8		44.2	44.0
		W	0.025	0.085		**W**	38.4		39.0	39.2
ψ	Prod. gap at $\bar{h}$ hours	**M**	0.033	0.044	Share bunching at full-time hours	**M**	44.2		44.1	44.1
		W	0.011	0.024		**W**	45.1		45.8	45.8
κ	Vacancy posting cost	**M**	0.071	0.071	Expected vac. cost / qrtly earnings*	**M**	14.0		14.1	14.0
		W	0.060	0.062		**W**	14.0		14.0	14.1
σ_ε	Match prod., standard dev.	**M**	0.054	0.057	Empl. separation rate	**M**	1.79		1.75	1.82
		W	0.106	0.120		**W**	2.63		2.59	2.74
δ	Separation shock	**M**	0.005	0.004	Share exogenous empl. separation*	**M**	25.0		25.0	25.0
		W	0.007	0.007		**W**	25.0		25.0	25.0
s_e	On-the-job search intensity	**M**	0.317	0.313	Job-to-job transition rate	**M**	1.71		1.72	1.71
		W	0.302	0.307		**W**	1.75		1.76	1.75
ω_1	Cost of working job 1	**M**	0.195	0.098	Employment rate	**M**	95.0		95.1	94.9
		W	0.196	0.082		**W**	93.5		93.6	93.2
ω_2	Cost of working job 2	**M**	0.038	0.041	Multiple jobholding share	**M**	6.57		6.57	6.61
		W	0.072	0.083		**W**	6.62		6.75	6.60

Notes: The table describes the model parameters (left panel) that provide the best fit to the data (right panel). M and W denote model and data moments for respectively men and women; $\mathcal{F}$ denotes the targeted Frisch elasticity of labor supply. The model period is set to be one month. The data moments except those marked with an asterisk are based data from the Current Population Survey for individuals aged 25 to 54 with some College or higher education; those marked with an asterisk are taken from the literature. All moments except the Frisch elasticity of labor supply and average hours per worker are expressed in percent.

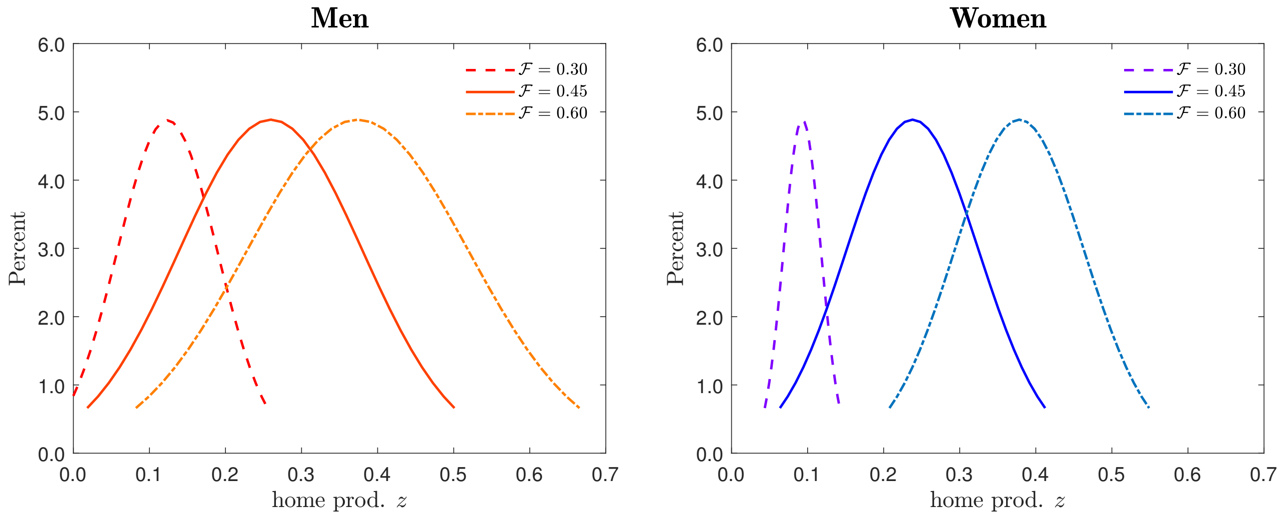

Figure C1: Distribution of home productivity z

Notes: The panels in this figure plot, for men and women, the cross-sectional distribution of worker's home productivity, z. In each panel, the lines refer to models calibrated to match three different values (0.30, 0.45, 0.60) of the Frisch elasticity of labor supply, $\mathcal{F}$.

To illustrate graphically the difference between the different calibrations (i.e., $\mathcal{F} = 0.30$, $\mathcal{F} = 0.45$, $\mathcal{F} = 0.60$), Figure C1 plots the distribution of home productivity, z, of each parameterization. A larger value of the Frisch elasticity shifts the mean and the dispersion of those distributions to the right. It also implies more persistence of the stochastic process.

To illustrate further the workings of the model, in Figure C2 we plot a reservation threshold called $\tilde{y}_d(y_1, z)$ that plays a key role in the model's equilibrium. $\tilde{y}_d(y_1, z)$ is defined by: $E(y_1, \tilde{y}_d(y_1, z), z) = E(y_1, z)$, i.e., it is the lowest value of match productivity of the second job y_2 for the worker to accept it. As can be seen $\tilde{y}_d(y_1, z)$ increases with both match productivity of the first job y_1 and home productivity z. When match productivity of the first job is higher, the worker has fewer hours to devote to the second job, which makes her more selective about the second job. The worker is also more selective when she is less willing to add more hours. This occurs when her own idiosyncratic home productivity is higher. Note that $\tilde{y}_d(y_1, z)$

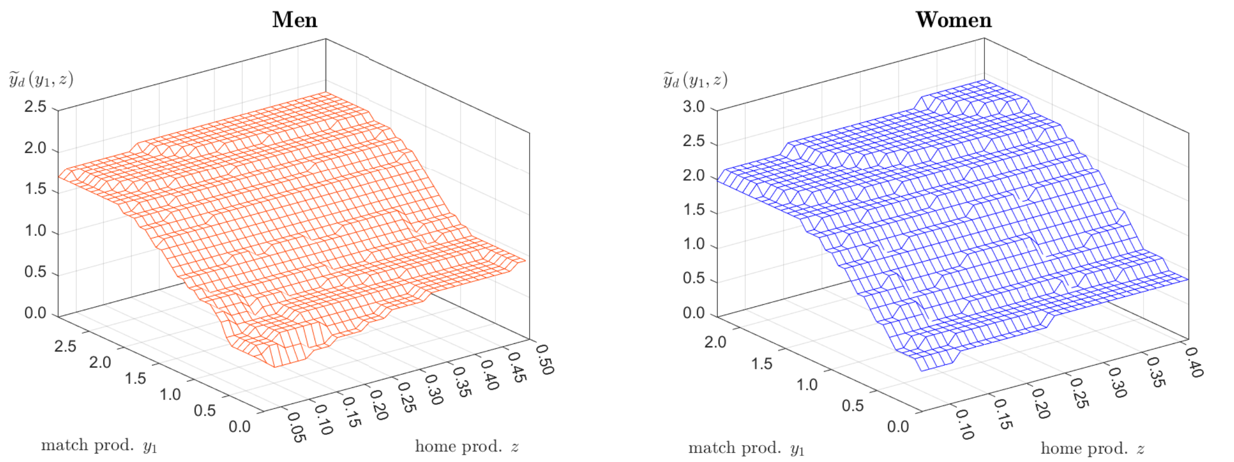

Figure C2: Reservation thresholds for multiple jobholding

Notes: The panels in this figure plot, for men and women, the reservation thresholds $\tilde{y}_d(y_1, z)$ that define the decision to take on a second job. $\tilde{y}_d(y_1, z)$ depends on match productivity of the first job, y_1, and the workers' home productivity, z. The underlying parameter values are those matching a Frisch elasticity of labor supply of 0.45.

describes the worker's decision when she is not switching to the outside employer, i.e., when the match productivity draw at the outside employer is $y_2 \leq y_1$.

Table C2 describes the calibrated parameter values of the model without multiple jobholding (with $\mathcal{F} = 0.45$ as a target for the Frisch elasticity). In Table 6 of the text, we summarized the main differences between the parameter values of this model and those of the full model with multiple jobholding. It is useful to note that in this version too, the model achieves a very good fit to the targeted data moments (last column of Table C2).

Table C2: Internally calibrated parameters: Model without multiple jobholding

Parameter	Description		Value	Targeted moment		Data	Model
γ	Curvature of $g(1-h)$	**M**	0.350	Frisch elasticity of labor supply*	**M**	0.45	0.45
		W	0.281		**W**	0.45	0.45
μ_z	Home prod., uncond. mean	**M**	0.249	Part-time empl. share	**M**	4.92	5.00
		W	0.228		**W**	17.6	17.3
ρ_z	Home prod., persistence	**M**	0.825	Full- to part-time trans. prob.	**M**	1.29	1.31
		W	0.946		**W**	3.13	3.10
σ_z	Home prod., standard dev.	**M**	0.120	Average hours per worker	**M**	43.8	44.2
		W	0.076		**W**	38.4	39.7
ψ	Prod. gap at $\bar{h}$ hours	**M**	0.040	Share bunching at full-time hours	**M**	44.2	44.2
		W	0.022		**W**	45.1	45.9
κ	Vacancy posting cost	**M**	0.075	Expected vac. cost / qrtly earnings*	**M**	14.0	14.0
		W	0.063		**W**	14.0	14.1
σ_ε	Match prod., standard dev.	**M**	0.043	Empl. separation rate	**M**	1.79	1.78
		W	0.101		**W**	2.63	2.63
δ	Separation shock	**M**	0.005	Share exogenous empl. separation*	**M**	25.0	25.0
		W	0.007		**W**	25.0	25.0
s_e	On-the-job search intensity	**M**	0.314	Job-to-job transition rate	**M**	1.71	1.73
		W	0.347		**W**	1.75	1.76
ω_1	Cost of working job 1	**M**	0.134	Employment rate	**M**	95.0	94.9
		W	0.119		**W**	93.5	93.5

Notes: The table describes the model parameters (left panel) that provide the best fit to the data (right panel). M and W denote model and data moments for respectively men and women. The model period is set to be one month. The data moments except those marked with an asterisk are based data from the Current Population Survey for individuals aged 25 to 54 with some College or higher education; those marked with an asterisk are taken from the literature. All moments except the Frisch elasticity of labor supply and average hours per worker are expressed in percent.